Introduction to Modern Physics : Special Relativity

Outlook : En route towards General Relativity

Reasons that call for a generalization of relativity

The principle of relativity as stated so far is only valid in inertial reference systems : i.e.: non-accelerated frames. This implies that at least one system of that sort exists, i.e. a system which is totally isolated, or in other words which is subjected to no forces. If we consider that gravity is an infinite range force, this may not be the case in any place of our universe. On top of that it is ethically unacceptable to admit that the form of the physics laws would differ if the reference frame were to be accelerated.

By definition, electromagnetic forces are correctly described in special relativity. They stem from the Maxwell equations which are Lorentz invariant. But this is not the case of the Newtonian gravitation force which is a classical mechanics concept. After 1905, Einstein tried to include gravity into special relativity but his findings contradicted the law of free fall in a gravitation field : in particular the vertical free fall speed would depend upon the horizontal speed of the body. This was one more reason to correct/extend special relativity.

Einstein paid a little more attention to Galileo Galilei's free fall law.

General principle of relativity

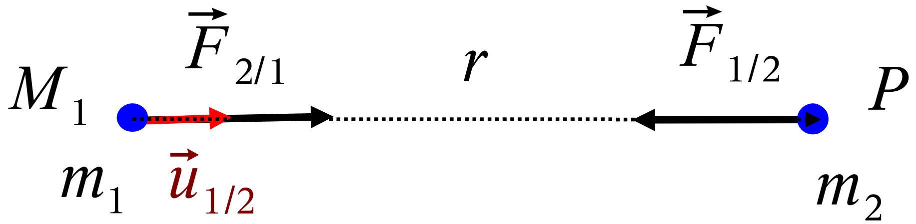
All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics.

At first sight, this may appear in contradiction with our immediate observations when we are subjected to acceleration, as we clearly feel when an acceleration is exerted or not, i.e. when a lift starts or stops : the higher the acceleration, the bigger our feeling is.

But Einstein understood that contrary to our first impressions, this observation precisely confirmed the principle of general relativity.

Gravitational field

Between two point-like masses m_1 and m_2 located at points M_1 and P separated by a distance r :



$$\vec{F}_{1/2} = -G\frac{m_1 m_2}{r^2}\vec{u}_{1/2}$$

$G = 6.6743\ 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

$\vec{g}(P) = -G\frac{m_1}{r^2}\vec{u}_{1/2}$ gravitational field – Property of space induced by m_1 located at M_1

$[g] = m\, s^{-2}$

$\vec{F}_{1/2} = m_2\vec{g}(P)$ If g is produced by a general mass distribution (distribution of point-like masses), its expression differs from the formula given above, but this relation remains correct.

The gravitational field is a more general concept than the gravitation force, like the electric and magnetic fields in electromagnetism. When a gravitational perturbation is induced, it propagates through the gravitational field at the limit velocity with respect to an IRS.

Equivalence Principle

If gravitation is the only interaction exerted on m_2 : $\vec{F}_{1/2} = m_2 \vec{g}(P) = m_2 \vec{a}(P) \Rightarrow \vec{g}(P) = \vec{a}(P)$

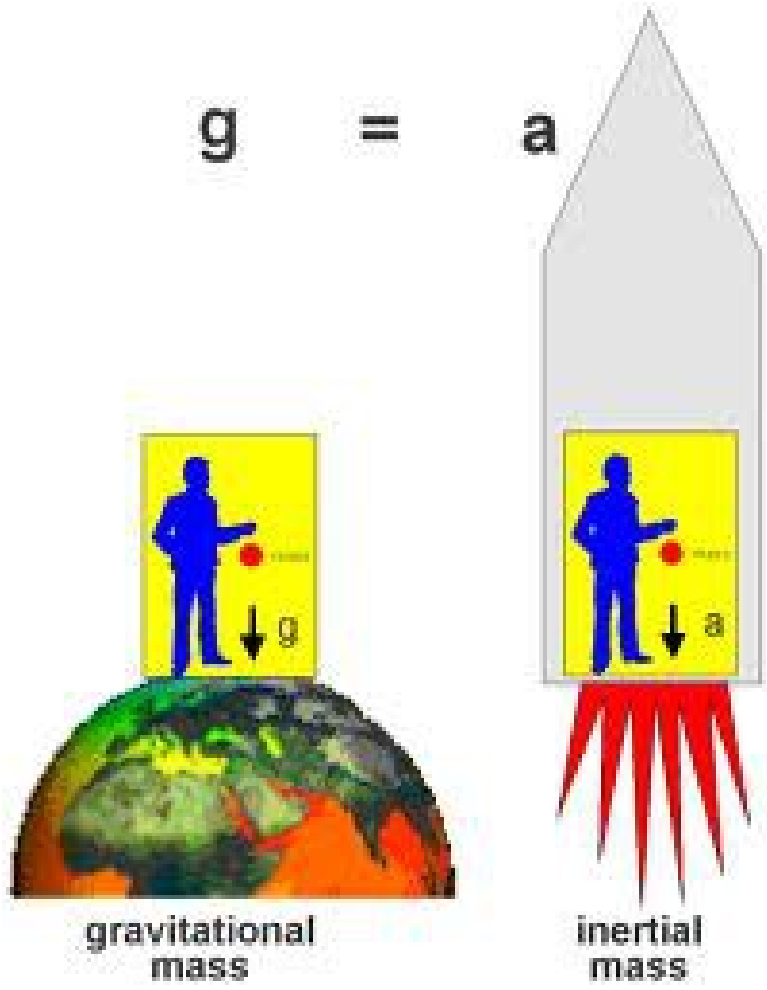
A gravitation field is locally equivalent to an acceleration field (Equivalence Principle of Einstein)
if the gravitational mass is equal to the inertial mass (experimentally verified with great precision)
(Equivalence Principle of Newton or Weak Equivalence principle)

In a closed and local (small) laboratory, no experiments can be performed that will distinguish between the effects of a gravitational field and the effects due to an acceleration field with respect to an inertial reference frame.

Conclusion : Going from an IRS to an accelerated frame is like adding gravity !

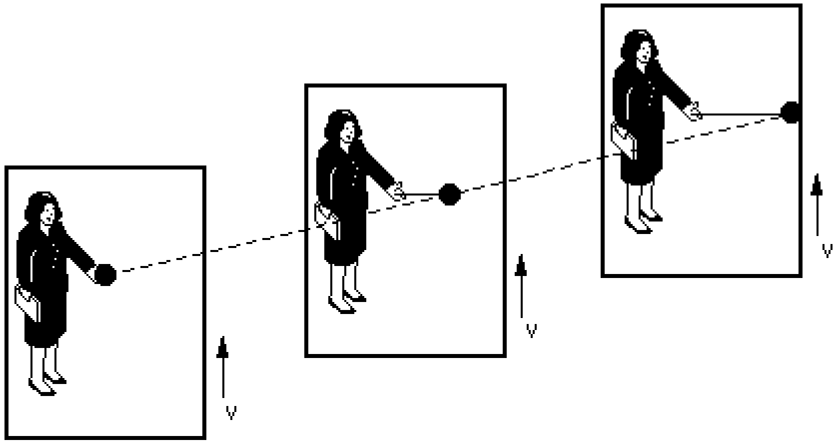
Inversely, removing gravity, e.g. in a local free falling laboratory is like going to an inertial reference frame.

Equivalence Principle

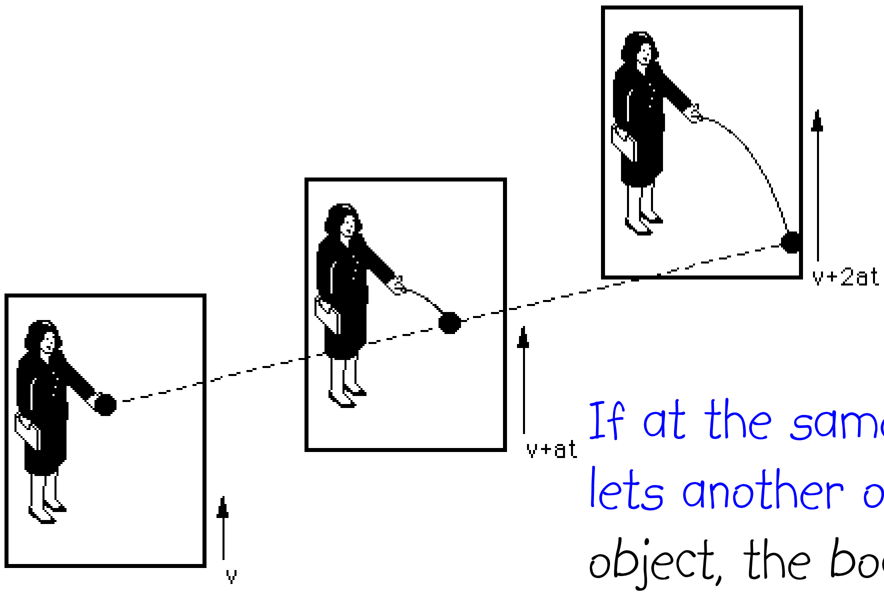


The rocket accelerates
with respect to an
Inertial Reference System

Free fall revisited in the context of the Equivalence Principle



The ball is horizontally thrown in an IRS moving at the constant speed v. Observed inside, it moves horizontally and hits the opposite wall. An external observer can see the ball moving along a straight and slanted line.



The ball is now horizontally thrown in a uniformly accelerated reference system. Observed inside, it moves downwards along a parabola. An external observer still sees the ball moving along a straight and slanted line.
The free fall is the same as in a fixed reference system subjected to a constant gravitational field.

If at the same time as the ball is released, the observer lets another object freely fall, with respect to this new reference object, the body will again horizontally move, removing gravity.

The strong equivalence Principle

The gravitational motion of a small test body depends only on its initial position in spacetime and on its initial velocity, and not on its constitution.

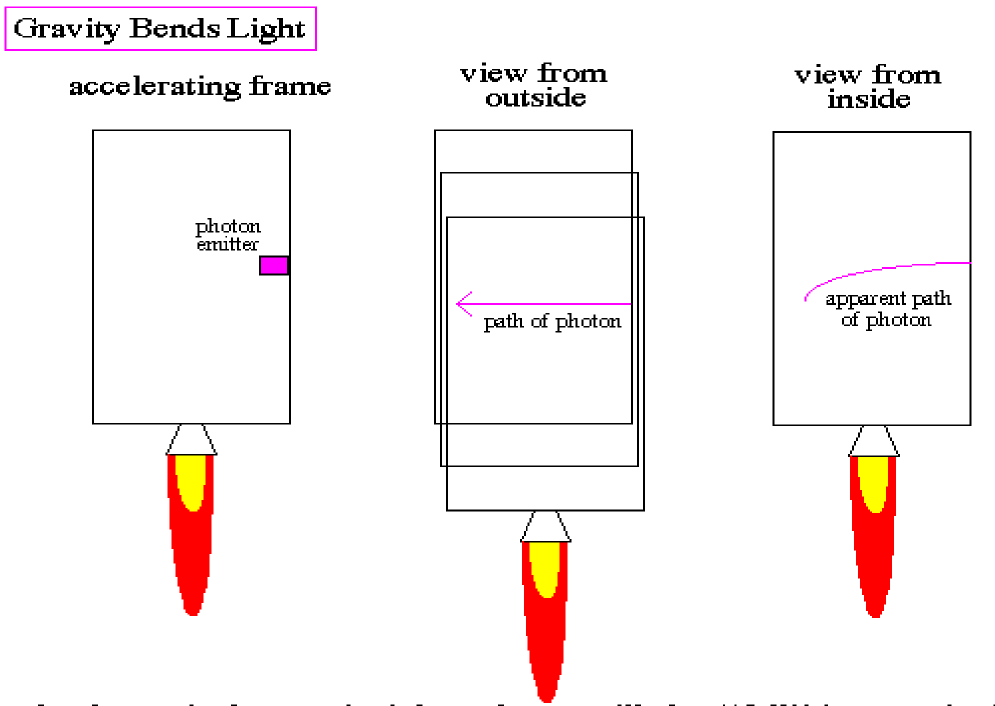
and

The outcome of any local experiment (be it gravitational or not) in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.

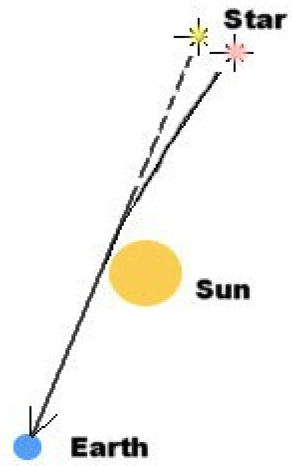
Einstein's theory of general relativity is the only theory of gravity that complies with the strong equivalence principle.

In other words, it says that a local freely falling RS is an inertial reference system. Then in a local freely falling reference system, the laws of special relativity apply.

A few physical consequences of the equivalence principle



by the equivalence principle, a photon will also "fall" in a gravitational field



Prediction of GR, angular deviation = 1.7 '' , confirmed by Arthur Eddington and Frank Dyson in 1919.

One may also notice that inside the spaceship, the velocity of light at the end of the trajectory is greater than that at the beginning. The second principle of SR is not valid anymore in an accelerated frame.

A few physical consequences of the equivalence principle

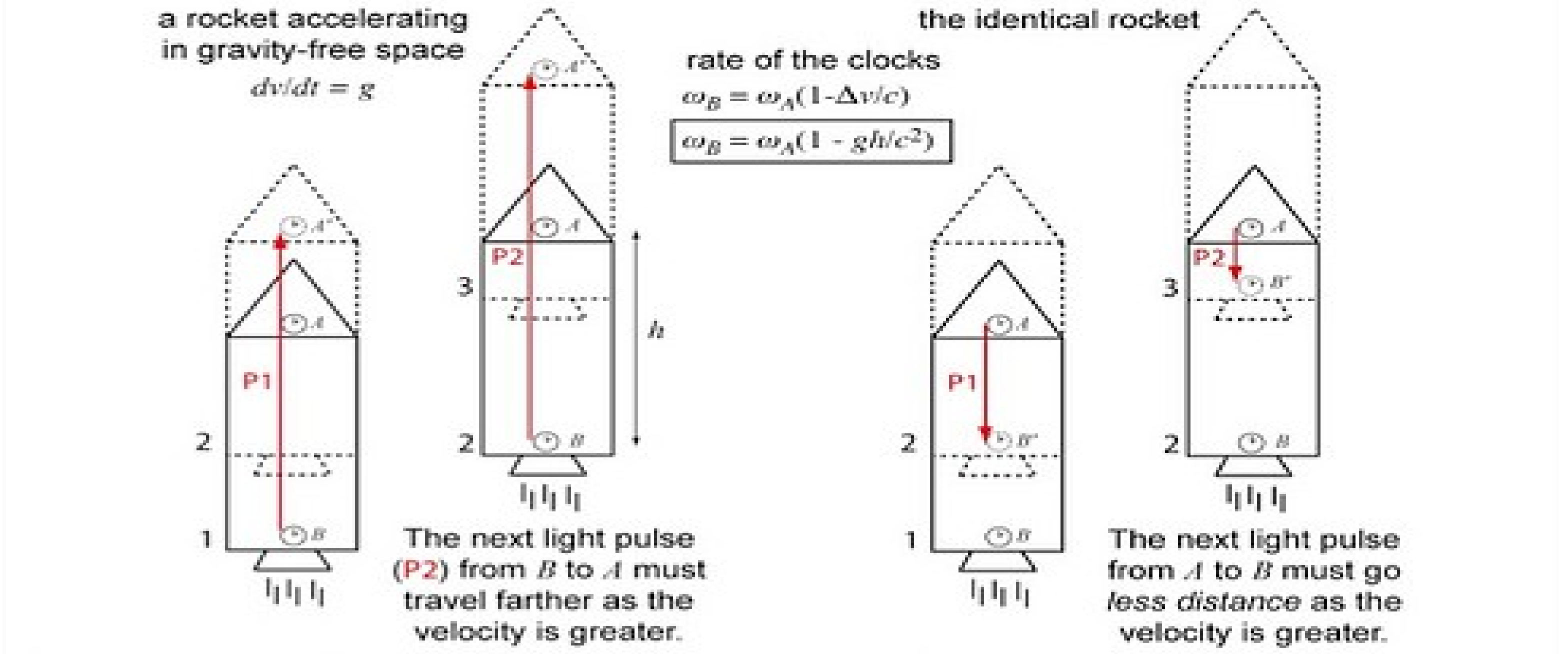


Figure 41 | Inertially accelerated ideal clocks A and B tick with flashes of light. The constancy of the speed of light implies that, over a given period of time, a free-falling inertial observer sees fewer flashes (clock ticks) arrive at clock A sourced from clock B than those that arrive at clock B sourced from clock A. Consequently, clock B ticks slower than clock A in an absolute sense.

As a consequence clocks subjected to gravity differently beat if they are located at different elevations. The slowest clock being the lowest one in the gravitational field.

Geometry of General Relativity

In special relativity, the invariant interval reads :

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

Metric : to compute the interval knowing the coordinates variations. This is a pure geometrical object.

In SR, spacetime is pseudo-Euclidean (Minkowski space), the metric always reduces to :

$$g = \eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In General Relativity, the interval expression reads the same, but Gaussian coordinates are now used instead of the Cartesian ones and the metric varies from point to point as a function of the mass content of the Universe. Spacetime is now a Riemannian manifold with an intrinsic curvature which is the manifestation of gravity. Gravity is not anymore a force but a pure geometrical phenomenon (hence Galileo's law of free fall).

If the mass distribution evolves, so do the Universe and its geometry.

The mathematics of GR are beyond the second university year level, but I encourage you to prolong your journey during the coming years, as GR is really one of the splendors of modern science (and also daily used in the GPS system).

Back to GPS

In the Earth gravitational field, the proper time elementary interval of a clock orbiting at the distance r from the Earth center is approximately given by :

$$d t_0 \simeq \left(1 - \frac{G M}{c^2 r} - \frac{v^2}{2 c^2}\right) d t$$

Earth mass

elementary time interval at r = infinity

gravitational correction

Special relativity correction due to the orbiting velocity v

Remembering that the GPS satellites orbit at 20400 km from the Earth surface, one can compute that the gravitational correction implies that the clocks located on the Earth lag by 48 µs behind the satellite clocks per day while the SR correction produces a delay of 7.8 µs per day of the satellite clocks with respect to the Earth clocks. The net correction is a delay of the ground clocks of 38.1 µs per day.

GPS atomic clocks need frequent synchronisations but their frequency is tuned slightly lower than the value of the ground clocks (10.22999999543 MHz instead of 10.23 MHz) to correct most of the relativistic delay.

To learn more :

-Special Relativity : A first encounter , Domenico Giulini, Oxford University Press

-Relativity , Albert Einstein , Penguin classics

-Relativité restreinte, Claude Semay et Bernard Silvestre-Brac, Dunod

-Relativité Générale, Aurélien Barrau & Julien Grain, Dunod

-On the Shoulders of Giants: The Great Works of Physics and Astronomy, edited with commentary by Stephen Hawking

-Histoire d'une grande idée : la relativité , Banesh Hoffmann