Introduction to Modern Physics : Special Relativity

Lecture 1 : Historical Preamble – Principles of Special Relativity – Principle of Global Positioning System (GPS)

Historical Preamble

- Classical mechanics prevailed as the physical science paradigm until the beginning of the XIXth century. Our infinite, steady state universe had a Euclidian geometry and was endowed with an absolute reference system : absolute coordinate frame + absolute time

- In 1801, Thomas Young's double-slit experiment set the beginning of wave optics. Light was then modeled as waves that propagate in a new medium : ether, that filled the universe, as light is observed to reach us from distant stars. The absolute reference system was bound to ether

- A few decades later, Michael Faraday postulated the existence of the electric and the magnetic fields. Taking this as a starting point, James Maxwell edified the theory of the electromagnetic fields : Maxwell equations. Electricity, magnetism, optics were now seen as different manifestations of a common physical reality. Ether bore the electromagnetic fields. Light was one species of a more general concept : electromagnetic waves, which were discovered by Heinrich Hertz in 1888.

Historical Preamble

- But the electromagnetic waves appeared to propagate at a constant velocity in vacuum (ether) whatever the velocities of their source or observer. This became one of the biggest scientific puzzles of the end of XIXth century.

- Some even thought that classical mechanics – a pure mathematically-based science - that once unified the terrestrial and astronomical mechanics, could even be considered as a different discipline from physics.

- The puzzle was progressively elucidated by Lorentz, Fitzgerald, Poincaré ... and finally the young Albert Einstein in 1905 in a famous article called : On the electrodynamics of moving bodies published in the German review Annalen der Physik.

- But as we will see, the solution proved once again that science is unique and indivisible

Basic physical questions :

Imagine yourself waking up after a rather long sleep in an isolated place with absolutely no external interactions : in other words somewhere in the universe, far from any visible astronomical bodies. And you try to answer those basic questions : this is a gedanken experiment.

- -Where are you ?
- -What time is it ?
- -What is your velocity ?
- -What is your acceleration ?

-Think also of a different observer that would be somewhere else in the universe in the same conditions of isolation

Why is it so ?

Why isn't it possible to define *absolute* positions in the universe, nor any *absolute* times, nor any *absolute* velocities ?

The answer can be guessed from the law of classical dynamics :

$$\vec{F}=m\vec{a}=m\frac{d^2\vec{r}}{dt^2}=m\frac{d\vec{V}}{dt}$$

if : $\vec{r}\rightarrow\vec{r}\,'=\vec{r}+\vec{r}_0$, $d^2\vec{r}\,'=d^2\vec{r}$ makes no differences, so any fixed coordinate frame leads the same results.

if : $\vec{V}\rightarrow\vec{V}\,'=\vec{V}+\vec{V}_0$, $d\vec{V}=d\vec{V}\,'$ makes no differences, so any constant velocity coordinate frame (inertial frame) leads the same results.

if : $t\rightarrow t\,'=t+t_0$, $d^2t=d^2t\,'$ makes no differences, so any regularly beating clock (an inertial clock) leads the same results.

But if the acceleration changes, then the force is modified and this can be observed. Think of your feeling when you stand in an elevator when it starts lifting you.

Inertial (Galilean) reference system :

Inertial frame : constant (and very often unknown) velocity frame

Inertial clock : regularly beating clock

Obviously an inertial reference system (IRS) is subjected to no external forces.

Beware that an inertial frame may be endowed with different coordinates systems : Cartesian, cylindrical, spherical coordinates ...

Exercise : do inertial reference systems exist in nature ? Try to find a few examples. If not what should be the good approach to define the physical laws ?

First principle of special relativity : also called Einstein's principle of relativity.

If no inertial systems can be distinguished, they are all equivalent. Any of them can be used to express the laws of physics.

Einstein postulated that :

All physical laws are the same in all inertial reference systems.

Note that only a single word changed with respect to the classical version of the principle of relativity : *All mechanical laws are the same in all inertial reference systems.*

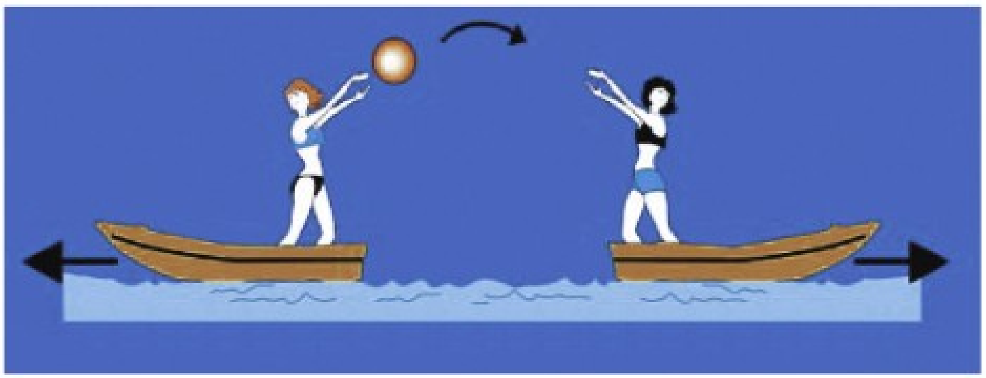
In a sense, Einstein conjectured that the laws of electromagnetism and mechanics could be reconciled in a common coherent framework.

As a consequence, no absolute motion exists and there's no preferred inertial reference system.

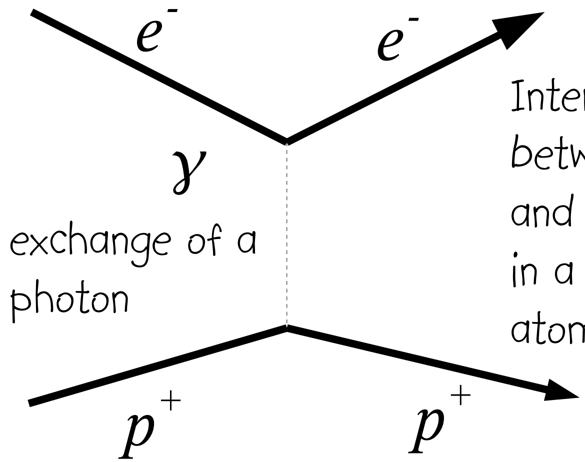
Action at a distance

All fundamental interactions (gravitation, electromagnetism, weak and strong interactions) proceed through action-at-a-distance forces.

Newton's *objection* : *That gravity should be innate*, inherent & *essential* to matter, *so* that one *body* may act upon another at a distance through a vacuum without the mediation of anything *else*, *by* & through which their action of force may *be* conveyed from one to another, is to me *so* great an *absurdity* that I *believe* no man who has in philosophical matters any competent faculty of thinking, can *ever* fall into it.



L'échange de bosons est responsable de la force



Interaction between an electron and a proton in a hydrogen atom

In modern physics, we consider that forces proceed through the exchange of mediators.

Causality and limit velocity

Physical phenomena are always time ordered processes. In other words, the cause should always strictly precede the effect. This is called causality.

In action-at-a-distance interactions, this implies that a limit (maximal) velocity should exist, otherwise interactions could be instantaneously conveyed at an infinite velocity, with the undesired consequence that the cause and the effect would be synchronous and undistinguishable. This would violate causality.

Later we will see that the limit velocity is the speed of massless particles in vacuum. As photons (light particles) are massless, it turns out that this is also the speed of light in vacuum. But all massless particles travel at this speed in vacuum.

Let us designate this limit velocity by c.

Second principle of Special Relativity : universality of limit velocity

Interaction signal traveling at the limit velocity
(think of light if you prefer)

As in general the observer ignores the velocity of the source, and since it does not know about its absolute speed, a logical and consistent statement that can be made is to postulate c constant, universal and independent of the source and the observer states of motion. This is the content of the second SR principle :

The modulus of the limit velocity in vacuum observed in an inertial reference system is universal and does not depend upon the state of motion of the source nor the observer.

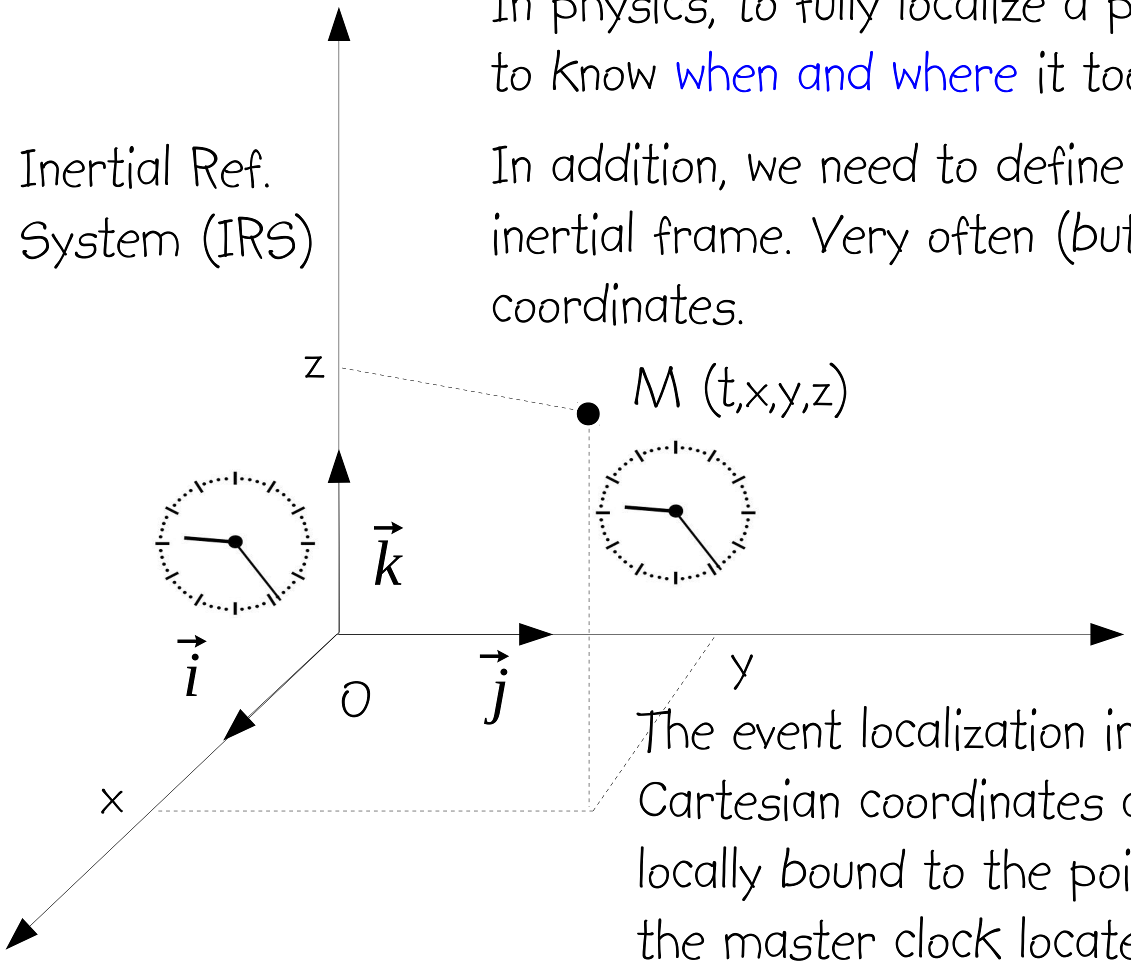
As c is universal, in 1983 it was set to a given value in the SI unit system : c = 299 792 458 m/s

Note that Einstein referred to light in his second principle, but as we already pointed out c is the velocity of all massless particles.

Event localization in the universe

In physics, to fully localize a particular event in the universe, one has to know when and where it took place in a given inertial ref. system.

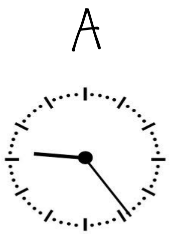
In addition, we need to define a coordinate system which is bound to the inertial frame. Very often (but not always) we use the Cartesian coordinates.



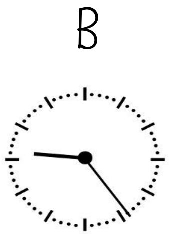
The event localization in the universe is then given by (x,y,z), the Cartesian coordinates and t, the time read on a clock which is locally bound to the point (x,y,z) and which is at rest with respect to the master clock located at the origin. This clock is synchronized with the master clock . The set of all these clocks (there's one per space point) forms a network of synchronized clocks.

Simultaneity

Definition : two events at spatially separated locations A and B are simultaneous, if and only if the local time readings of synchronized clocks at A and B are identical.



$$t_A = t_B$$



This definition leads to :

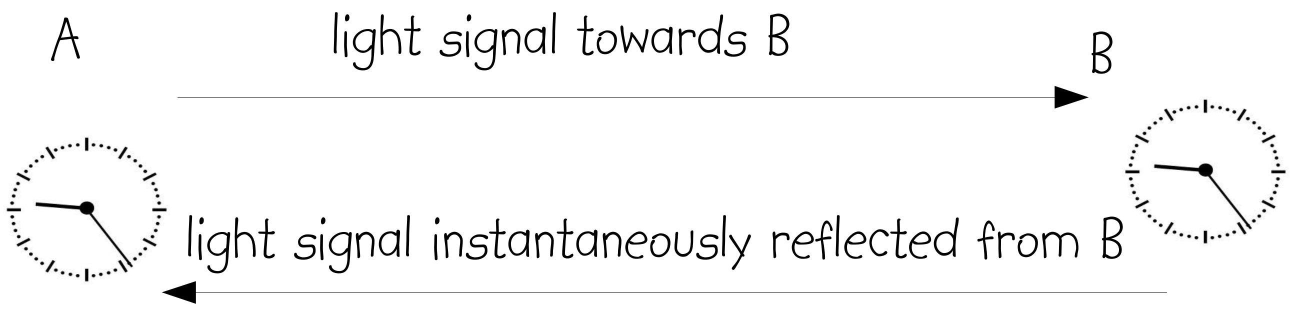
-every event is simultaneous to itself

-if p is simultaneous to q then q is simultaneous to p

-if p is simultaneous to q and q is simultaneous to r, then p is simultaneous to r

The important point to notice here is that simultaneity is defined with respect to a network of synchronized clocks which are at rest in a particular IRS.

Clock synchronization



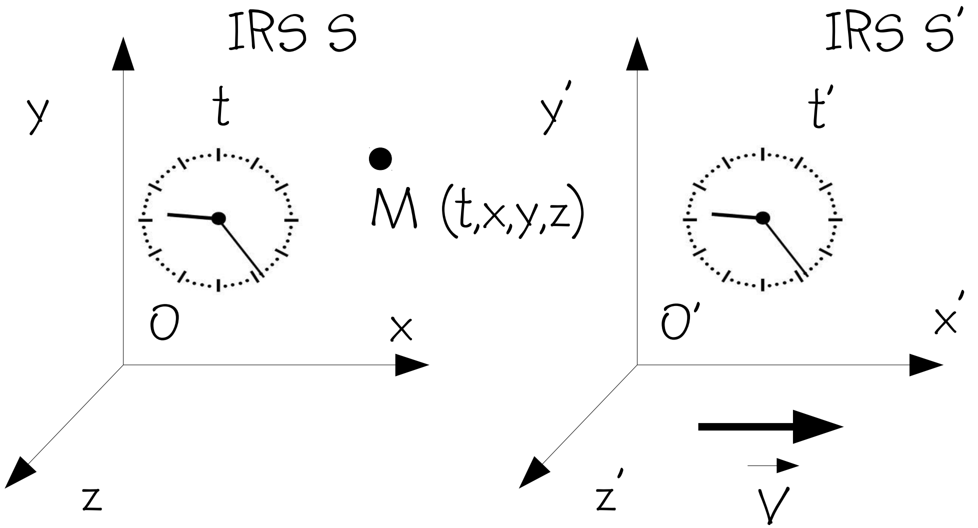
Clocks A and B are at rest with respect to a given IRS

If t_1 is the time measured by A at which the signal is sent and t_2 , the time measured by A at which it comes back from B, the propagation time is given by ½ $(t_2 - t_1)$. A may then send a command signal to B (using electromagnetic waves) with the instruction to set the time of B to t (the time at which the command is sent) + ½ $(t_2 - t_1)$, the time it will take for the command to reach B.

The same procedure may be repeated with all inertial clocks. Conceptually, it may be considered that any space point may be equipped with such a synchronized clock.

Note that the clocks need to beat at exactly the same frequency to maintain the synchronization at all time.

Special Relativity / Classical Mechanics



S' moves with respect to S with a constant velocity V directed towards (o,x)

It's not difficult to show that in classical mechanics, the coordinates of M in S' are given by :

$t'=t$ (the clocks of O and O' may be synchronized when they coincide, and subsequently all clocks of S and S' are synchronized to O and O', respectively and forever)

$x'=x-Vt$

$y'=y$

$z'=z$

This is called the Galilean transformation.

Now let us assume that M travels at the velocity of light in S along (o,x), then

$x = c\ t,$ and $x' = ct - V\ t = (c - V)\ t = (c - V\ )\ t'$,

S' would observe that M travels at $c - V$, which contradicts the limit velocity universality principle.

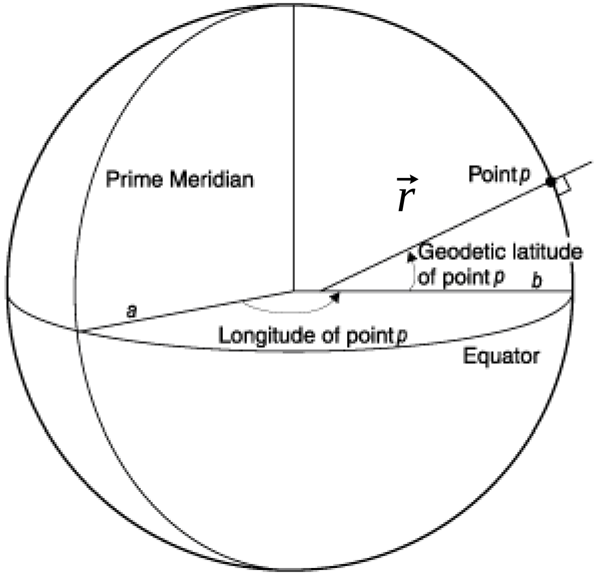
Special Relativity / Classical Mechanics

But the Galilean Transformation (GT) is indeed experimentally observed at low velocities. So the SR transformation should reduce to a GT at low speed and progressively diverge when nearing the limit velocity.

In other words, classical mechanics should be the limit behavior of special relativity at low velocities.

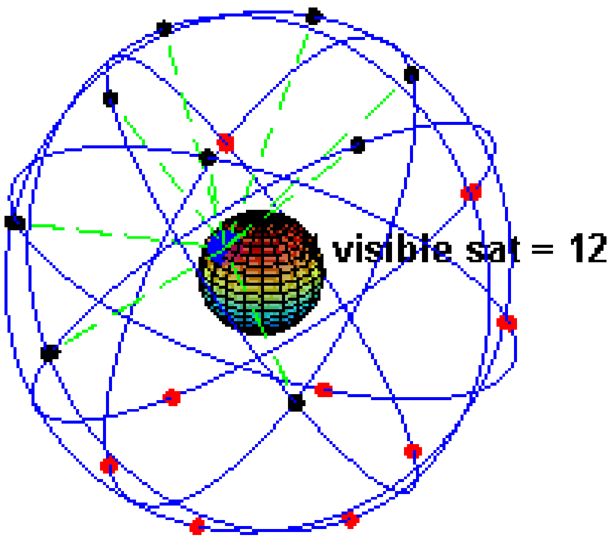
It is then not surprising that the first experimental disagreement with classical mechanics was observed with light and electromagnetic waves.

Global Positioning System (GPS)



To localize an event near the earth surface, we need to measure in a given IRS the time at which it happened, its latitude, its longitude and its altitude : (t,r,theta,phi) in spherical coordinates.

The global positioning system directly uses the limit velocity universality principle to determine the event coordinates.



A constellation of satellites orbit at a distance of 20 200 Km from the earth surface. Their orbits and number are set so as to guarantee that each terrestrial point simultaneously views a minimum of 4 satellites.

5 terrestrial stations are in charge of precisely localizing all satellites and of synchronizing their atomic clocks.

Global Positioning System (GPS)

Every 30 s, each GPS satellite broadcasts electromagnetic wave signals (L1= 1575.42 MHz , L2=1227.6 MHz) containing t_i , the emission time, and $\vec{r}_i$, the position coordinates of the satellite at the emission time.

A terrestrial GPS receiver simultaneously records a set of at least 4 equations w.r.t. at least 4 satellites :

$$d_i = |\vec{r} - \vec{r}_i| = c(t - t_i)$$

c=299 792 458 m/s

where : $\vec{r}$ and t are the unknown time and coordinates of the receiver.

Since there's at least 4 independent equations, the GPS receiver computer can determine the 4 unknown quantities : t, altitude, latitude, longitude

We will also see later, that the synchronization of the satellite clocks is regularly needed (many times per day) to correct the relativistic time drift.

Home work :

Are the GPS satellites and the GPS receivers inertial reference systems ?

What is the needed timing precision of the GPS to localize a terrestrial point with a precision of 10 m ?

What is the ultimate precision needed to observe the motion of the tectonic plates ?

Learn about the modern definition of a second, the international time system UTC (Universal Time Coordinated) and the modern definition of a meter.

Learn about the definition of a kilogram.

To learn more :

- Modern Physics for Scientists & Engineers, Stephen Thornton & Andrew Rex

- Physique Moderne, Stephen Thornton & Andrew Rex (traduit par R. Taillet & L. Villain)

- Special Relativity : A first encounter , Domenico Giulini, Oxford University Press

- Introduction à la Relativité : Johann Collot, http://lpsc.in2p3.fr/collot

- Relativité restreinte, Claude Semay et Bernard Silvestre-Brac, Dunod

- The principle of relativity, Dover, New York

- Albert Einstein's special theory of relativity, A. Miller, Springer

- On the Shoulders of Giants: The Great Works of Physics and Astronomy, edited with commentary by Stephen Hawking