Interaction of particles with matter

A brief review of a few typical situations is going to greatly simplify the subject.

Mean free path of a particle, i.e. average distance travelled between two consecutive interactions in matter :

$$\lambda = \frac{1}{\sigma\, n}$$

where :

σ total interaction cross-section of the particle

n number of scattering centers per unit volume

example : $n = \frac{\rho N_A}{M}$ for a monoatomic element of molar mass M and specific mass ρ .

N_A Avogadro number

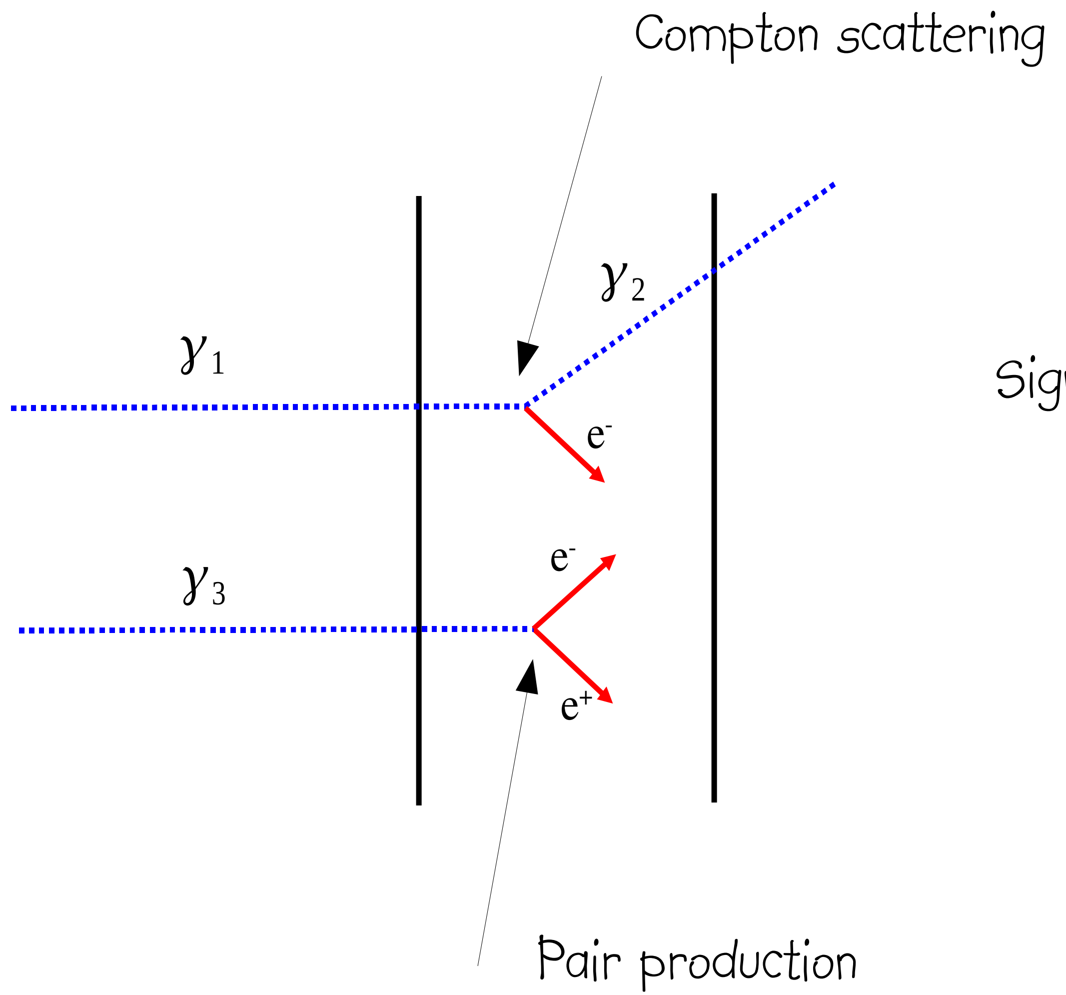
Electromagnetic interaction : $\lambda \leqslant 1\,\mu\mathrm{m}$ (charged particles)

Strong interaction : $\lambda \geqslant 1\,\mathrm{cm}$ (neutrons ....)

Weak interaction : $\lambda \geqslant 10^{15}\,\mathrm{m} \simeq 0{,}1$ light year (neutrinos)

A practical signal (>100 interactions or hits) can only come from electromagnetic interaction

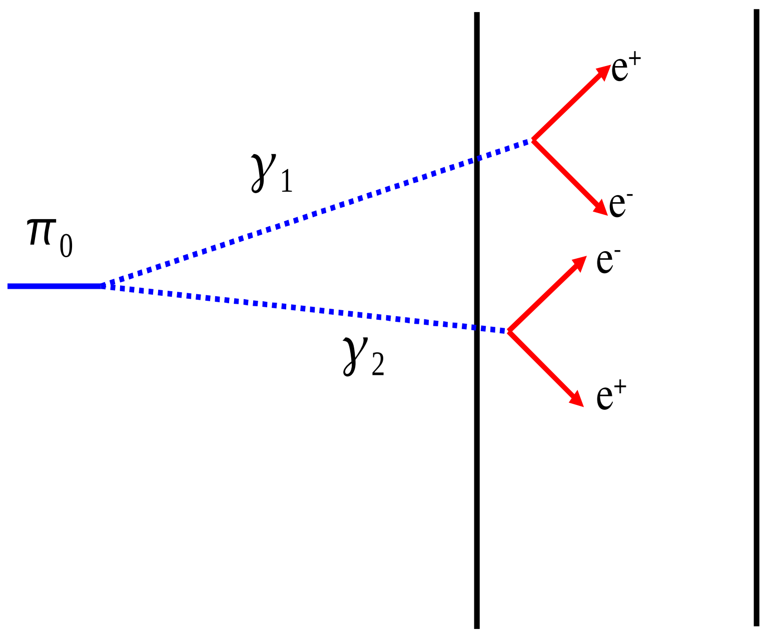
Particle detection proceeds in two steps : 1) primary interaction 2) charged particle interaction producing the signals typical examples : photon detection



Signal is induced by electrons

neutral pion detection :

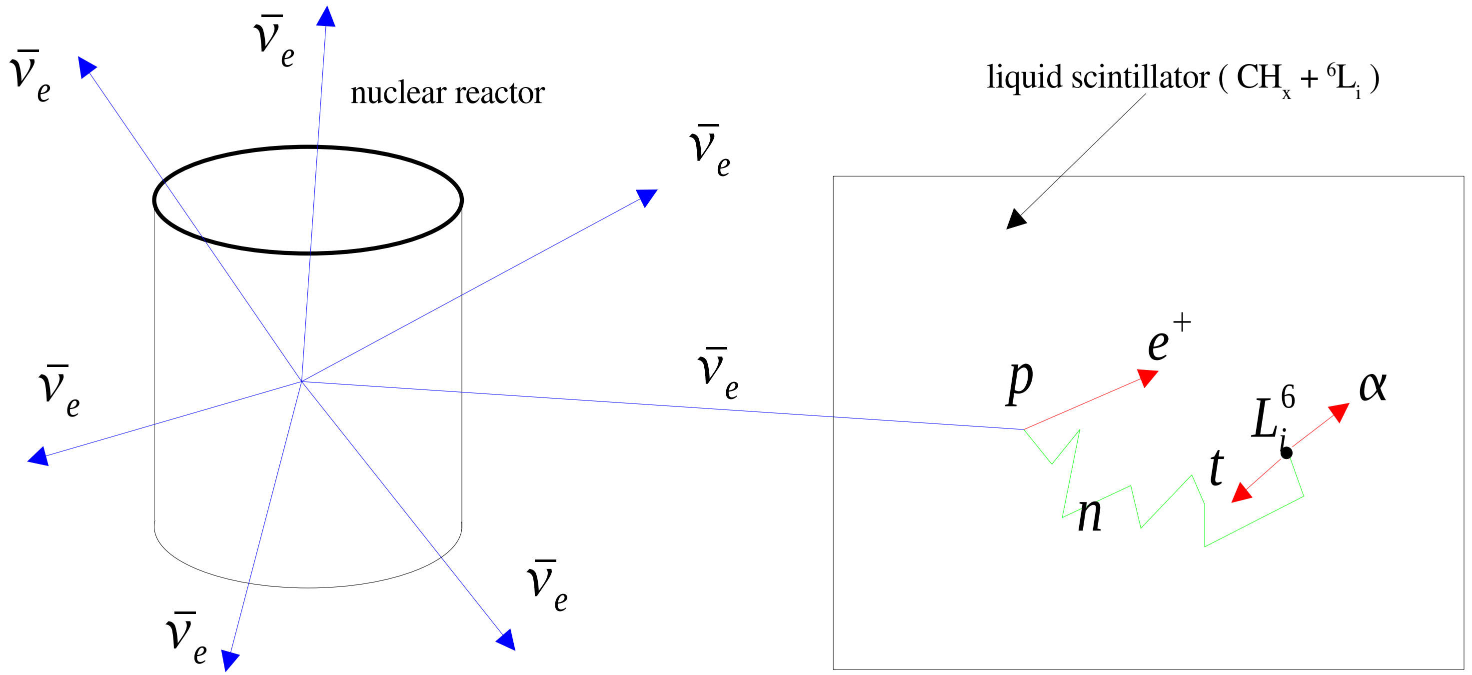
A π^0 decays into two photons with a mean lifetime of 8.4 10^{-17} s.



neutrino detection :

A 2800 MW nuclear power station produces 130 MW of neutrinos !

A detector of 1 m^3 located 20 m away from the reactor core can detect 100 neutrinos/h.



$\epsilon_\nu \sim 0$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$n_{th} + L_i^6 \rightarrow \alpha + t + 4{,}8\,MeV$$

Charged particles produce light in the target scintillator

Interaction of charged particles with matter

Ionization and excitation are the dominant processes producing energy loss.

Particle P of Z charge state

Excitation : $P^{(Z)} + \mathrm{atom} \rightarrow \mathrm{atom}^* + P^{(Z)}$ followed by : $\mathrm{atom}^* \rightarrow \mathrm{atom} + \gamma$

Ionization : $P^{(Z)} + \mathrm{atom} \rightarrow \mathrm{atom} + e^- + P^{(Z)}$

Ionization + excitation : $P^{(Z)} + \mathrm{atom} \rightarrow \mathrm{atom}^* + e^- + P^{(Z)}$

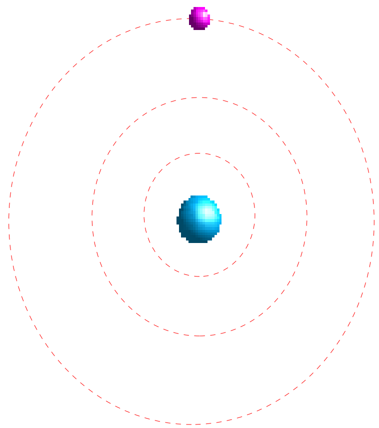
Maximal kinetic energy transferred to an ionized electron :

m_0 $\vec{v}$

$\vec{p}=\gamma m_0 \vec{v}$

E

m$_e$



atome

Z

$\beta=\frac{v}{c}$

$\gamma=(1-\beta^2)^{-1/2}$

$\beta=(1-\gamma^{-2})^{1/2}$

Hypothesis : $V > \langle v_e \rangle = Z\,\alpha\, c$, speed of deepest atomic orbit electrons where α is the fine structure constant : α = 1/137 .

One may show that : $$T_e^{max}=E_e^{max}-m_e=\frac{2\,m_e\beta^2\gamma^2}{(E_{CM}/m_0)^2}$$ (In natural units , c=ħ=1)

where : $$E_{CM}=(m_0^2+m_e^2+2\,m_e E)^{1/2}$$ total energy in center-of-mass frame

Two cases :

$m_0 \gg m_e$, i.e. the incoming particle is not an electron and if its energy is not too big

$$(E_{CM}/m_0)^2 = \left(\frac{m_0^2}{m_0^2} + \frac{m_e^2}{m_0^2} + \frac{2 m_e E}{m_0^2}\right) \simeq 1 \qquad \text{with} \qquad E = \gamma m_0$$

$$\frac{2\gamma m_e}{m_0} \ll 1$$

proton $E_p < 50$ GeV , muon $E_\mu < 500$ MeV

then : $$T_e^{max} = E_e^{max} - m_e = 2 m_e \beta^2 \gamma^2$$

$m_0 = m_e$ the incoming particle is an electron

$$T_e^{max} = (E - m_e)$$

If the incoming particle is not an electron then in practice $m_0 \gg m_e$.

Stopping power of heavy particles by excitation and ionization in matter.

Average energy loss by a charged particle (other than an electron) in matter.

Bethe and Bloch formula

(see Nuclei and particles, Émilio Segré, W.A. Benjamin and Principles of Radiation Interaction in Matter and Detection, C. Leroy and P.G. Rancoita, World Scientific)

Stopping power or mean specific energy loss

charge of incoming particle

Z of medium

density effect correction at high energy

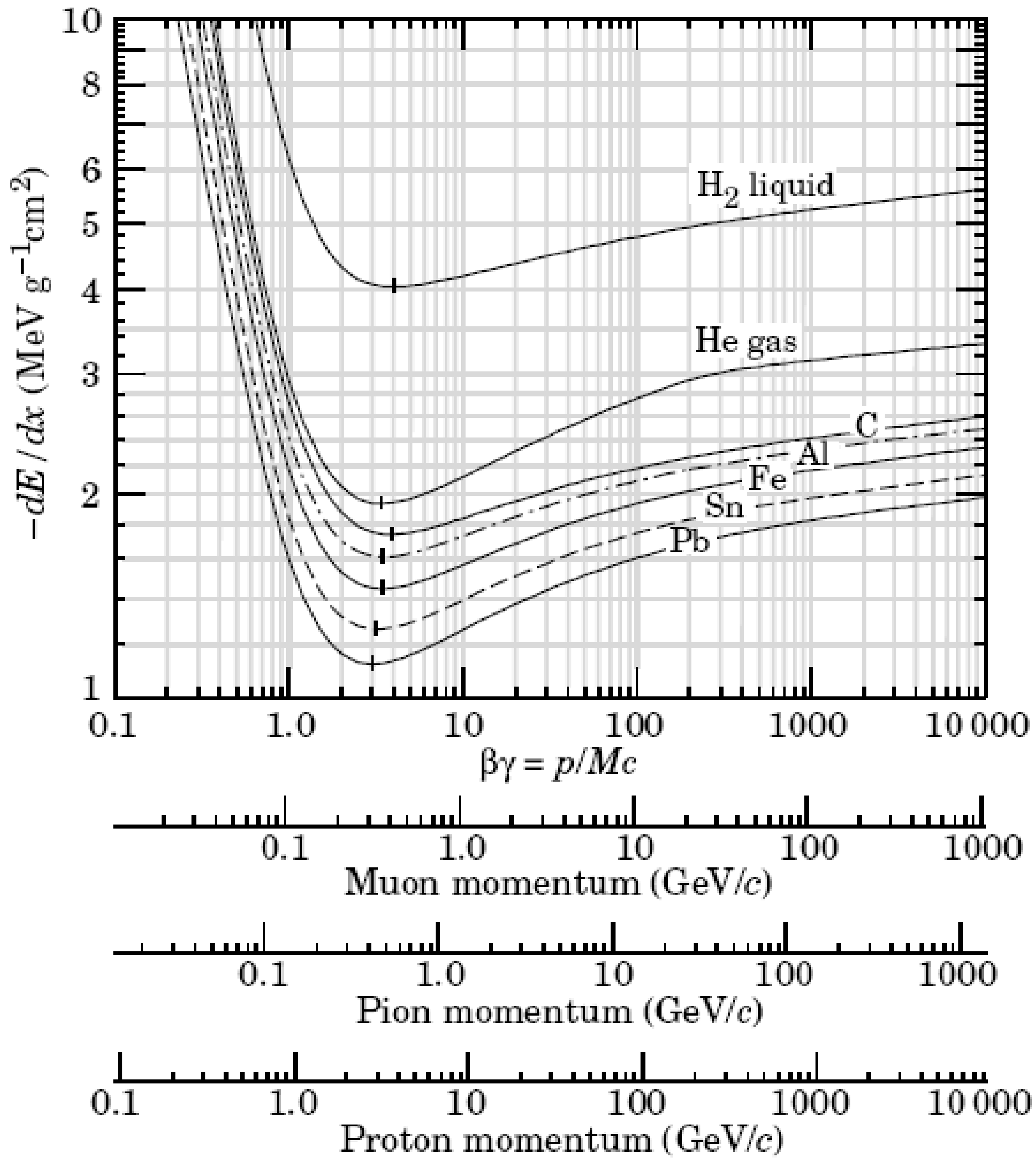
$$-\left(\frac{dE}{dx}\right)\left[\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\right] = \frac{0.3071}{A(\mathrm{g\ mol^{-1}})}\frac{z^2\ Z}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{2m_e\beta^2\gamma^2T_e^{max}}{I^2}\right)-\beta^2-\frac{\delta(\gamma\beta)}{2}-\frac{C_e}{Z}\right)$$

Atomic shell correction at low energy

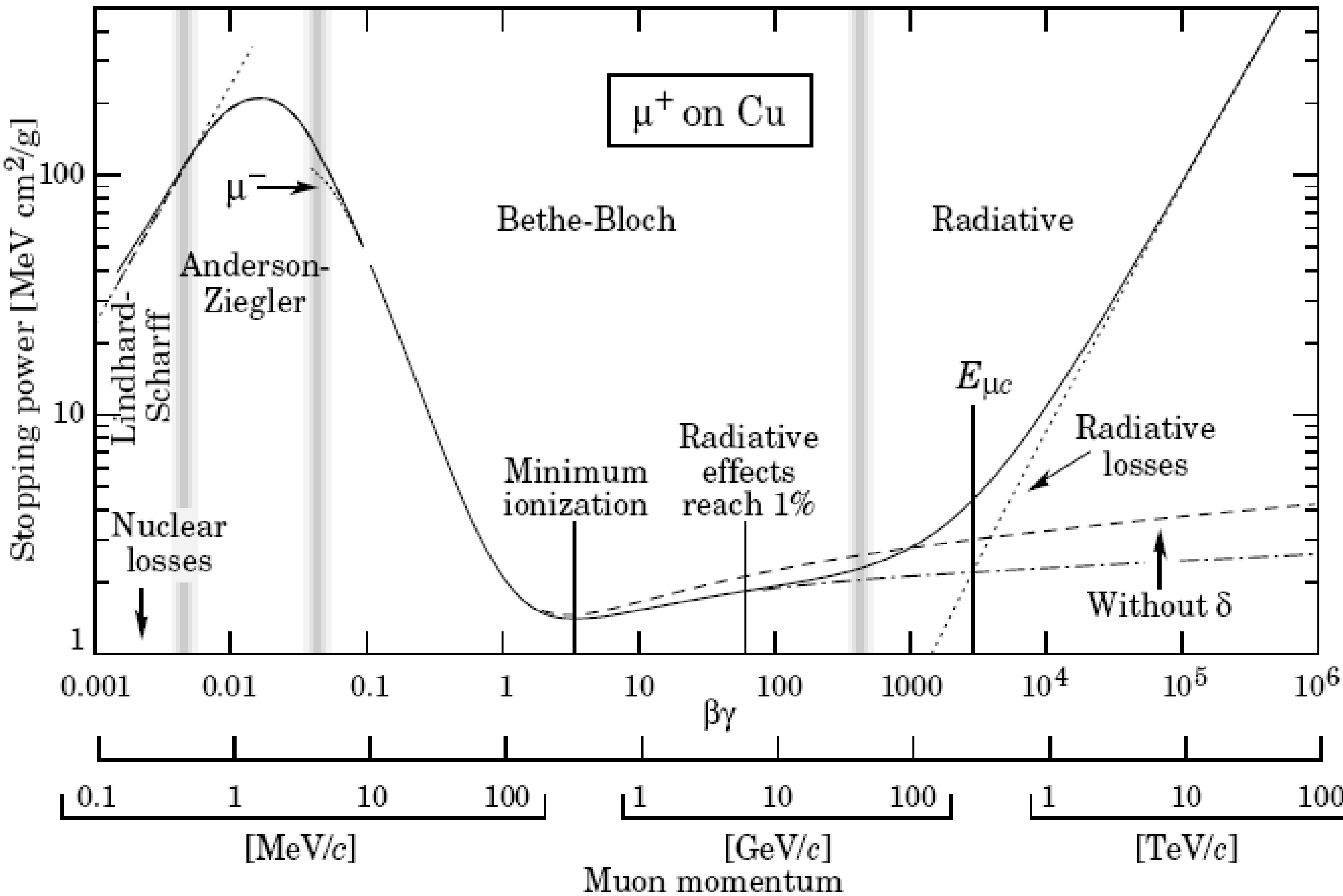
mean excitation energy

Atomic mass of medium

Surface mass density of medium $dx = \rho\ dl$ (or mass thickness of medium)



H2 liquid
He gas
C
Al
Fe
Sn
Pb
−dE/dx (MeV g−1cm2)
10
8
6
5
4
3
2
1
0.1
1.0
10
100
1000
10 000
βγ = p/Mc
Muon momentum (GeV/c)
Pion momentum (GeV/c)
Proton momentum (GeV/c)


μ⁺ on Cu
Stopping power [MeV cm²/g]
100
10
1
μ⁻
Bethe-Bloch
Radiative
Anderson-
Ziegler
Lindhard-
Scharff
Nuclear
losses
Minimum
ionization
Radiative
effects
reach 1%
Eμc
Radiative
losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10⁴
10⁵
10⁶
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

few remarks :

- for βγ < 1 : $-\frac{dE}{dx} \sim \beta^{-5/3}$ non relativistic particles

- for βγ ~ 4 : $-\frac{dE}{dx}$ is minimal over a large energy plateau . A particle in this state is called a minimum ionizing particle (MIP)

In media composed of light elements : $-\frac{dE}{dx}^{MIP} \simeq 2 \ \frac{\text{MeV}}{\text{g cm}^{-2}}$

- for βγ > 4 : relativistic increase of $-\frac{dE}{dx}$ as $\ln(\gamma)$ with is tempered by -δ/2 correction.

- I : mean excitation and ionization energy , I = 15 eV for atomic H and 19.2 eV for H_2

I = 16 $Z^{0.9}$ eV for Z > 1

At low energy : $\frac{2\gamma m_e}{m_0} \ll 1$ $T_e^{max} = 2 m_e \beta^2 \gamma^2$

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right] = \frac{0.3071}{A(g)} \cdot \frac{z^2 Z}{\beta^2}\left[\ln\left(\frac{2 m_e \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2} - \frac{C_e}{Z}\right]$$

Stopping power of electrons by ionization and excitation in matter.

Incoming and outgoing particles are identical.
Energy transfer is bigger .

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=\frac{0.3071}{A(g)}\cdot\frac{Z}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{T\,m_e\beta^2\gamma^2}{2I^2}\right)+\frac{1}{2\gamma^2}\left(1-(2\gamma-1)\ln(2)\right)+\frac{1}{16}\left(\frac{\gamma-1}{\gamma}\right)^2\right]$$

Kinetic energy of incoming electron : $T=(\gamma-1)m_e=E-m_e$

Stopping power of positrons by ionization and excitation in matter.

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=\frac{0.3071}{A(g)}\cdot\frac{Z}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{T\,m_e\beta^2\gamma^2}{2I^2}\right)-\frac{\beta^2}{24}\left(23+\frac{14}{\gamma+1}+\frac{10}{(\gamma+1)^2}+\frac{4}{(\gamma+1)^3}\right)\right]$$

When a positron comes to a rest it annihilates : $e^+ + e^- \rightarrow \gamma\,\gamma$ of 511 KeV each

A positron may also undergo an annihilation in flight according to the following cross section :

$$\sigma(Z,E)=\frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

Stopping power of a compound medium

$$\frac{dE}{dx} \approx \sum_i f_i \left.\frac{dE}{dx}\right|_i$$

$$f_i = \frac{m_i}{m} \quad , \quad \sum_i m_i = m$$ where f_i is the massic ratio of element i

$\left.\frac{dE}{dx}\right|_i$ is the stopping power of element i

Bremsstrahlung : electromagnetic radiative energy loss

A decelerated or accelerated charged particle radiates photons.
The mean radiative energy loss is given by :

fine structure constant = 1/137

medium atomic number

incoming particle energy

$$-\frac{dE}{dx}^{rad}\left(\frac{MeV}{g/cm^2}\right)=\frac{0.3071}{A(g)}\frac{\alpha}{\pi}Z^2 z^2\left(\frac{m_e}{m}\right)^2\frac{E}{m_e}\ln\left(\frac{183}{Z^{1/3}}\right)$$

incoming particle mass

incoming particle charge state

The mean radiative energy loss of a particle of charge z and mass m is a function of the mean radiative energy loss of an electron :

$$\frac{dE}{dx}^{rad}(z,m)=\left(\frac{m_e}{m}\right)^2 z^2\ \frac{dE}{dx}^{rad}(e^-)$$

Electrons are much more sensitive to this effect.

For an electron and taking into account the Bremsstrahlung radiation induced by atomic electrons :

classical radius of electron : $r_e = \alpha / m_e$

$$-\frac{dE^{rad}}{dx}(e^-) = 4\alpha N_A \frac{Z(Z+1)}{A} r_e^2 E \ln\left(\frac{183}{Z^{1/3}}\right)$$

which can be rewritten as :

$$-\frac{dE^{rad}}{dx}(e^-) = \frac{E}{X_0}$$

where X_0 is the medium radiation length

then over a path x in the medium, the mean radiated energy of an electron reads :

$$E^{rad}(e^-) = E(1 - e^{-x/X_0})$$

where x is expressed in cm or g/cm^2

and :

$$X_0(g/cm^2) = \frac{716.4\ A(g)}{Z(Z+1)\ln\left(\frac{287}{Z^{1/2}}\right)}$$

In a compound medium :

$$X_o = \left[\sum_i \frac{f_i}{X_0^i}\right]^{-1}$$

where f_i and X_0^i are the mass ratio and the radiation length of element i respectively.

Critical energy : energy at which the ionization stopping power is equal to the mean radiative energy loss of electrons

$$\frac{dE}{dx}^{rad}(E_c) = \frac{dE}{dx}^{ionization}(E_c)$$

$$E_c = \frac{610\ \ MeV}{Z+1.24}$$ for liquids and solids

$$E_c = \frac{710\ \ MeV}{Z+0.92}$$ for gas

medium	***Z***	***A***	X_0 ***(g/cm²)***	X_0 ***(cm)***	E_C ***(MeV)***
hydrogen	1	1.01	63	700000	350
helium	2	4	94	530000	250
lithium	3	6.94	83	156	180
carbon	6	12.01	43	18.8	90
nitrogen	7	14.01	38	30500	85
oxygen	8	16	34	24000	75
aluminium	13	26.98	24	8.9	40
silicon	14	28.09	22	9.4	39
iron	26	55.85	13.9	1.76	20.7
copper	29	63.55	12.9	1.43	18.8
silver	47	109.9	9.3	0.89	11.9
tungsten	74	183.9	6.8	0.35	8
lead	82	207.2	6.4	0.56	7.4
air	7.3	14.4	37	30000	84
silica (SiO_2)	11.2	21.7	27	12	57
water	7.5	14.2	36	36	83

Electron-positron pair production

At very high energy, direct electron-positron pair production may play an important role.

$$-\frac{dE}{dx}^{pair} = b_{pair}(Z, A, E)\ \ E$$

Energy loss by photo-nuclear interaction :

example : electro-dissociation of deuteron $\qquad e^- + d \rightarrow n + p + e^-$

$$-\frac{dE}{dx}^{\gamma nucl.} = b_{\gamma nucl.}(Z, A, E)\ \ E$$

Total stopping power :

$$\frac{dE}{dx}^{tot} = \frac{dE}{dx}^{ionization} + \frac{dE}{dx}^{rad} + \frac{dE}{dx}^{pair} + \frac{dE}{dx}^{\gamma nucl.}$$

which could also be written as :

$$-\frac{dE}{dx}^{tot} = a(Z, A, E) + b(Z, A, E)\ E$$

where a(Z, A, E) is the ionization term and b(Z, A, E) the sum of the Bremsstrahlung, the pair production and the photo-nuclear terms .

Multiple scattering through small angles

A charged particle traversing a medium is deflected many times by small-angles essentially due to Coulomb scattering in the electromagnetic field of nuclei.

This effect is well reproduced by the Molière theory. On both x and y, the angular deflection θ^{proj} of a particle almost follows a Gaussian which is centered around 0 :

$$(\theta^{space})^2=(\theta_x^{proj})^2+(\theta_y^{proj})^2$$

$$\theta_{rms}^{proj} = \frac{1}{\sqrt{2}}\theta_{rms}^{space}$$

$$P(\theta^{proj})\,d\theta^{proj}=\frac{1}{\sqrt{2\pi}\,\theta_0}\,e^{-\frac{1}{2}\left(\frac{\theta^{proj}}{\theta_0}\right)^2}\,d\theta^{proj}$$

$$P(\theta^{space})\,d\Omega=\frac{1}{2\pi\theta_0^2}\,e^{-\frac{1}{2}\left(\frac{\theta^{space}}{\theta_0}\right)^2}\,d\Omega$$

$$\theta_0=\frac{13{,}6\,MeV}{\beta p}\,z\sqrt{\frac{x}{X_0}}\,\left(1+0{,}038\ln\left(\frac{x}{X_0}\right)\right)$$

p particle momentum

X_0 radiation length

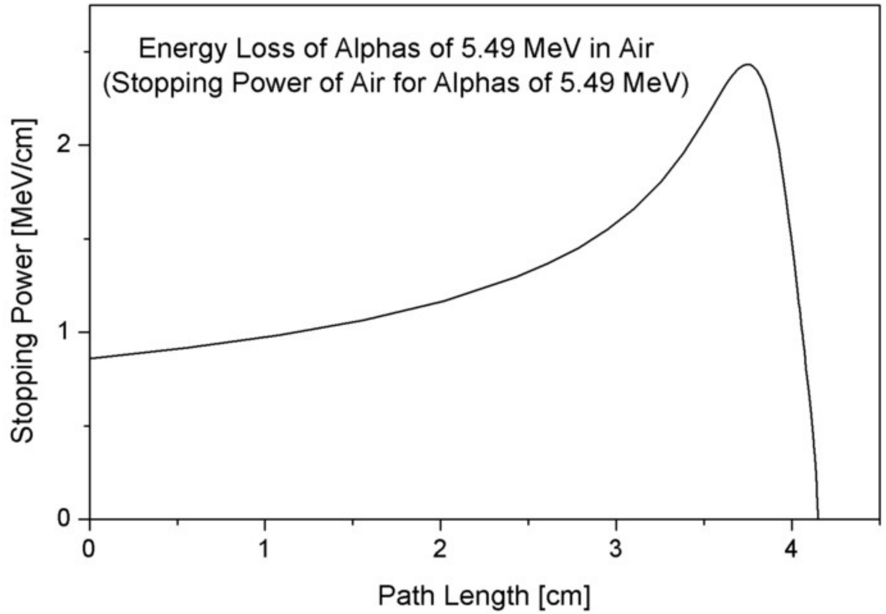
x medium thickness

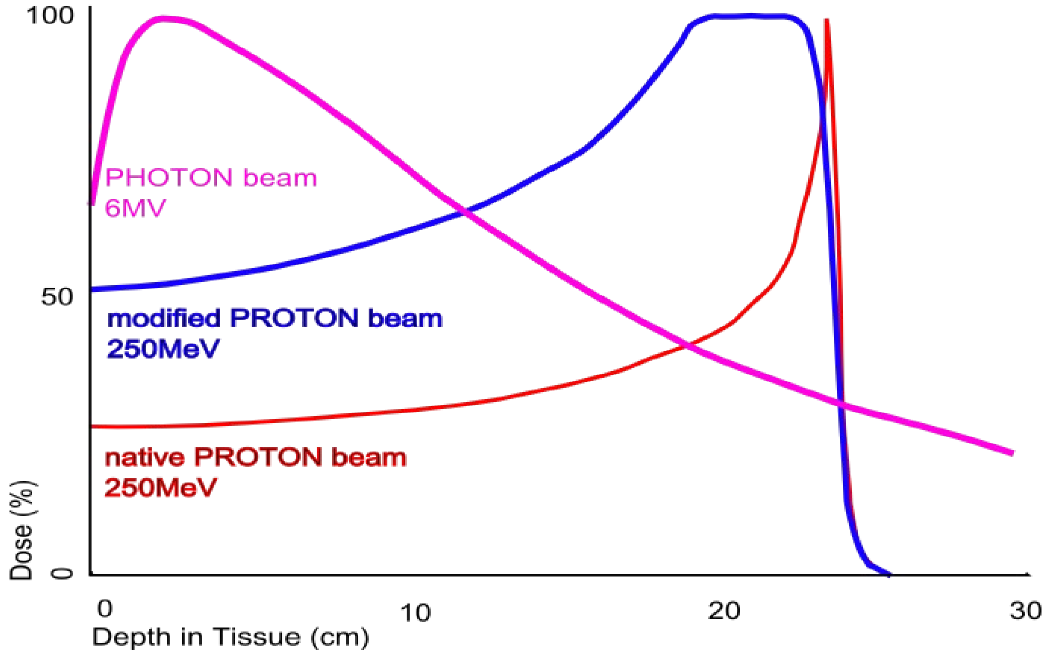
z charge state of incoming particle

Large deflection angles are more probable than what the Gaussian predicts. This results from Rutheford scattering of heavy particles off nuclei.

Particle Range in matter

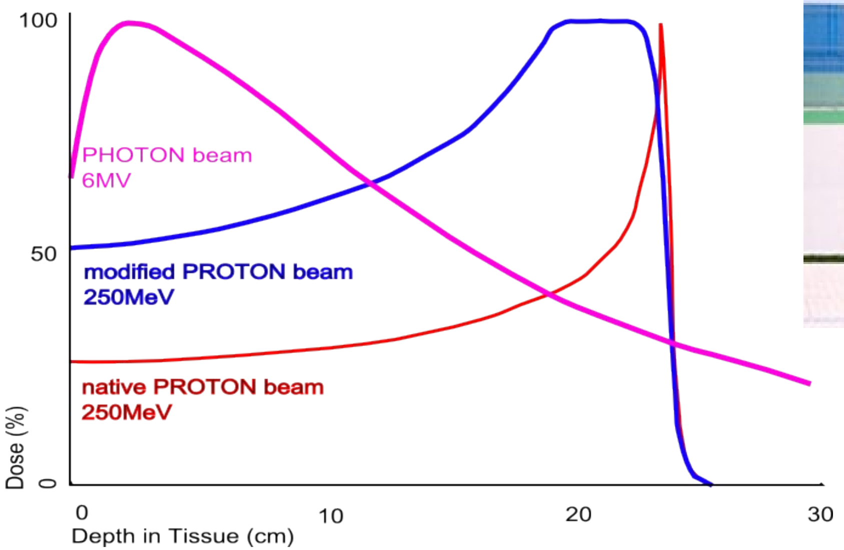
If the medium is thick enough, a particle will stop with an increasing stopping power at low energy ($\beta^{-5/3}$).





This stopping power profile is used in protontherapy for treating cancerous tumors

Wanjie Proton Therapy Center in Zibo (Shandong Province)



cyclotron delivering 230 MeV protons to treat cancerous tumors

As the result of the stochastic behavior of particles interacting in matter, it is not possible to enunciate a perfect definition of a reproducible particle range.

Continuous slowing down approximation range :

$$R(T_0)=\int_0^{T_o} \frac{dT}{\frac{dE}{dx}(T)}$$

where T_0 is the incident kinetic energy of the particle.

In practice, the integration is carried out down to 10 eV .

We also use the mean range <R> which corresponds to the distance at which half of the initial particles have been stopped.

If T> 1 MeV , $R \approx \langle R \rangle$.

If only ionization and excitation are used to calculate R(T) (valid for heavy particles with energies < 1 GeV), the following relationship can be used :

$$R_b(M_b, z_b, T_b) = \frac{M_b}{M_a} \frac{z_a^2}{z_b^2} R_a(M_a, z_a, T_b \frac{M_a}{M_b})$$

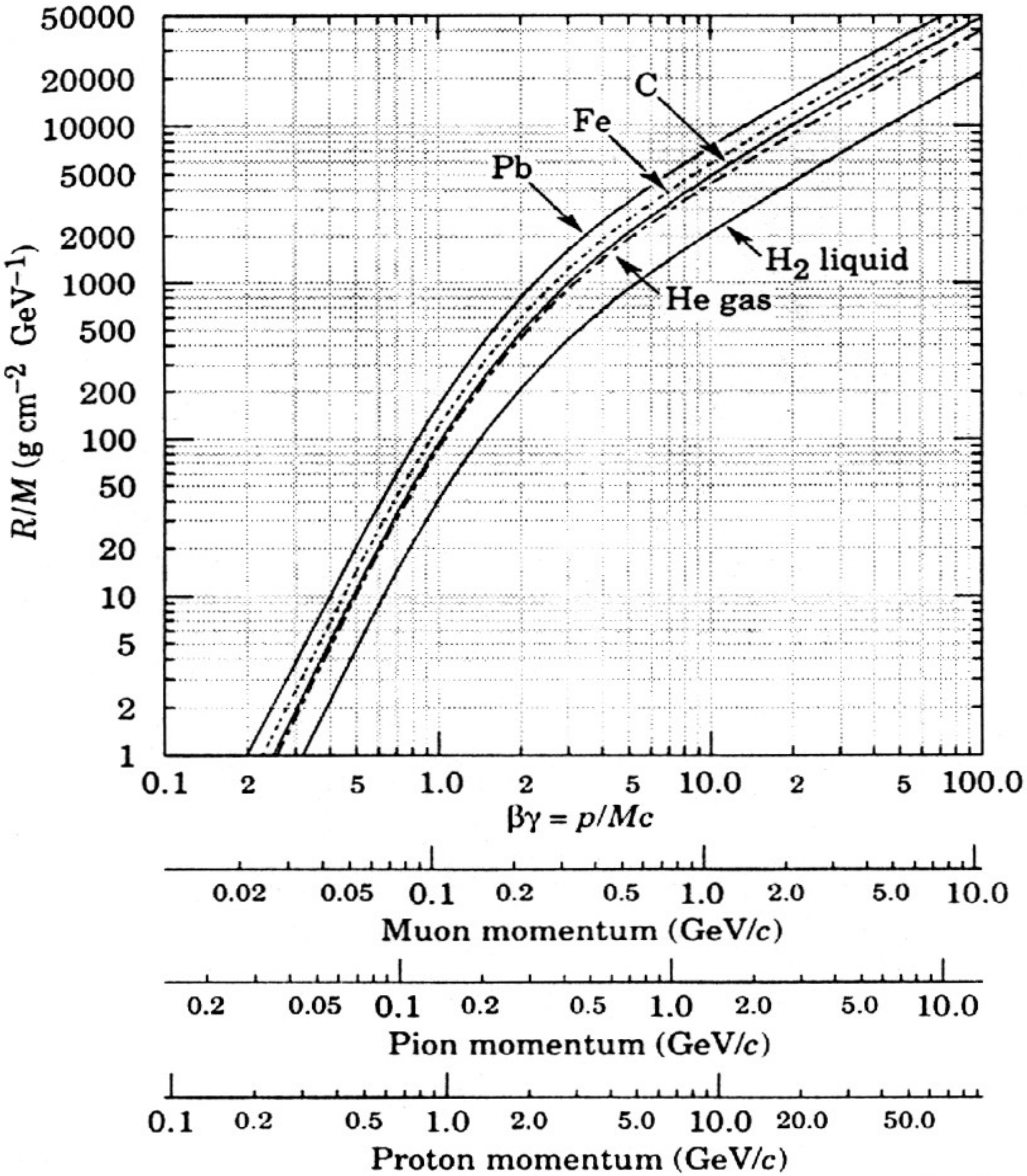
Particle a with z_a , M_a

Particle b with z_b , M_b and kinetic energy T_b

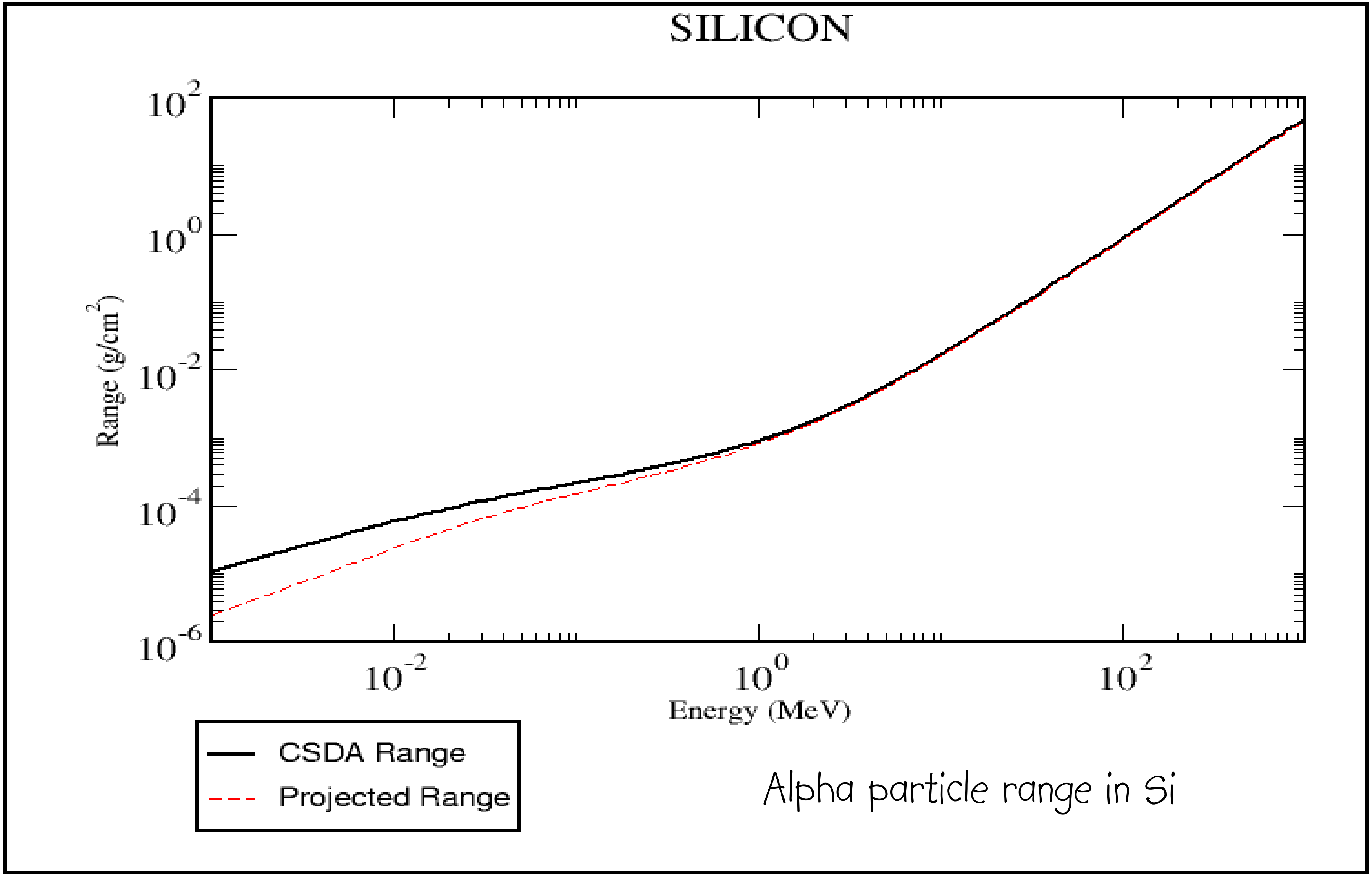
One may also write for a particle of mass M and charge state z carrying a kinetic energy T_0 :

$$R(M, z, T_0) = \frac{M}{z^2} h(T_0/M)$$

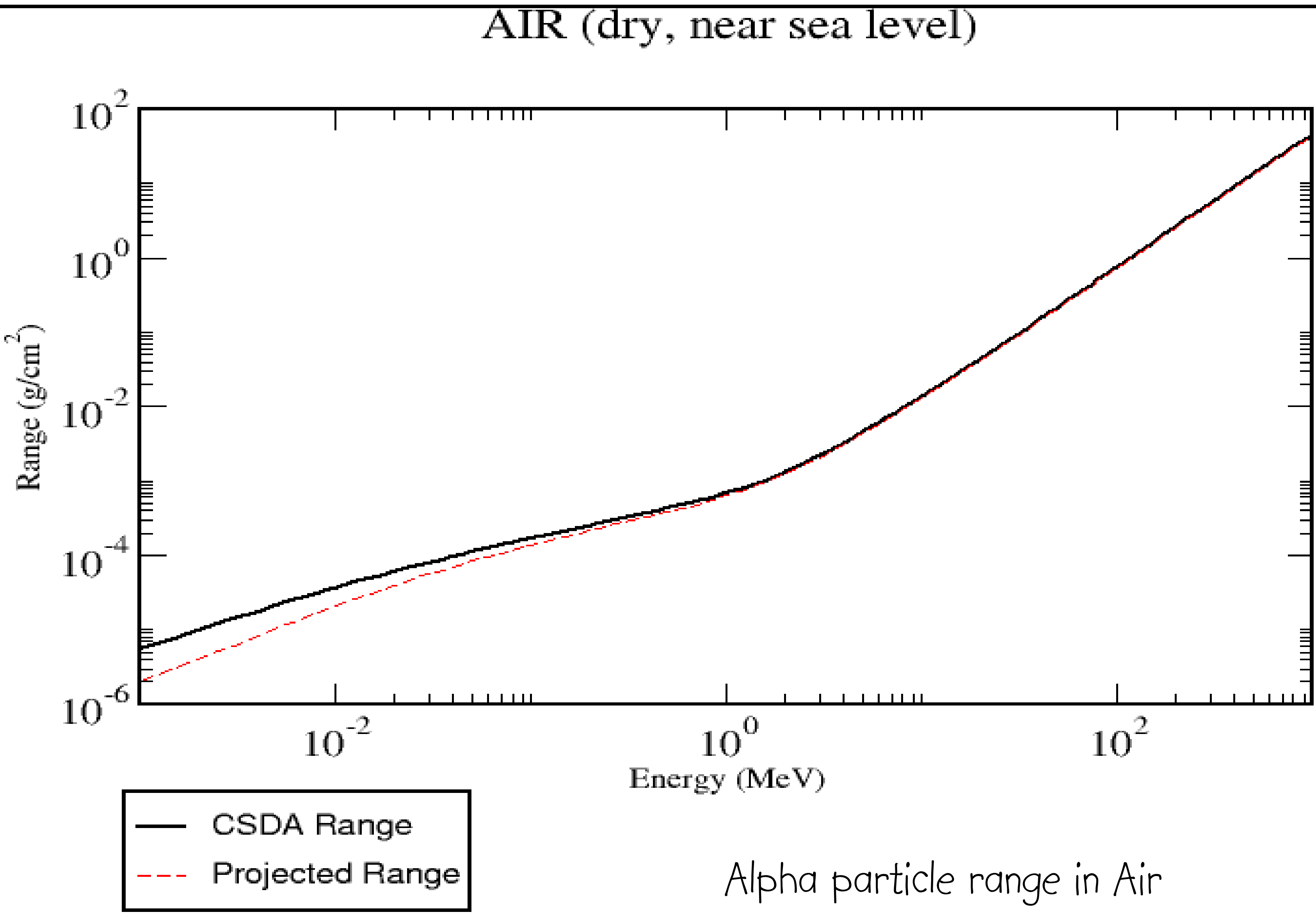
where h is a universal function of the medium (Z, A and I fixed).



universal h function = R .



http://physics.nist.gov/PhysRefData/Star/Text



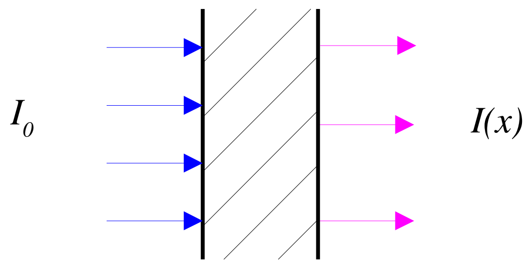
Alpha particle range in Air

Interactions of photons with matter :

Photons are indirectly detected : they first create electrons (and in some cases positrons) which subsequently interact with matter.

In their interactions with matter, photons may be absorbed (photoelectric effect or e^+e^- pair creation) or scattered (Compton scattering) through large deflection angles.

As photon trajectories are particularly chaotic, it is impossible to define a mean range. We then proceed with an attenuation law :



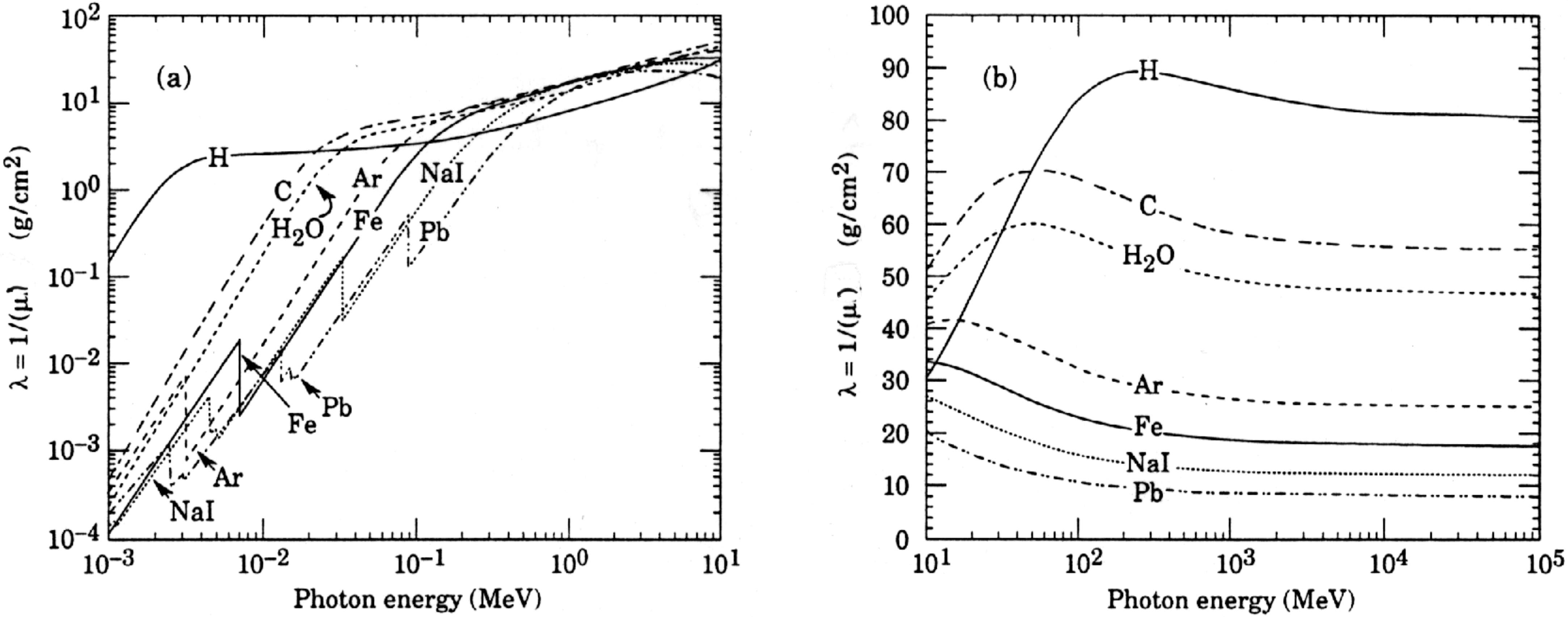
$$I(x) = I_0 e^{-\mu x}$$

$$\mu = \frac{N_A \sigma_{tot}}{A}$$

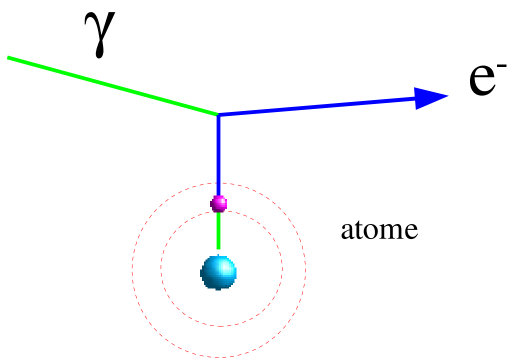
where :

- I_0 is the initial photon beam flux
- $I(x)$ is the photon beam flux exiting the layer of thickness x
- x surface density of the layer (g/cm^2)
- μ is the mass attenuation coefficient (cm^2/g)
- σ_{tot} is the total photon cross-section per atom

Photon mean free paths in different media : $\lambda = \dfrac{1}{\mu}$

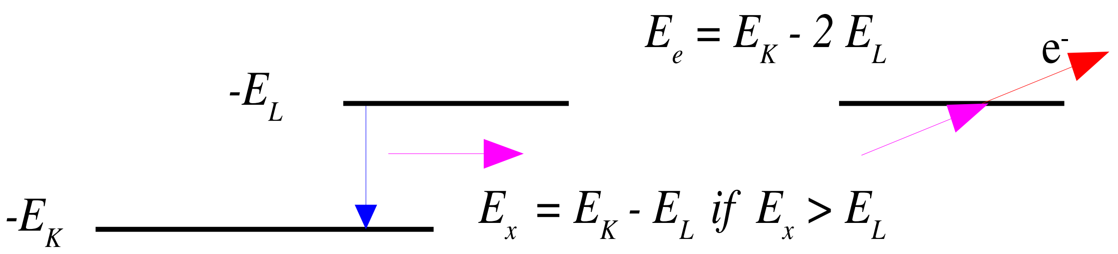


Photoelectric effect :



Because of their proximity to the nucleus, electrons of the deepest shells (K,L,M...) are favored.

Following the emission of a photoelectron, the atom reorganises leading to the production of X rays or Auger electrons.



Production scheme of an Auger electron

Photoelectron energy :

$E_e = E_\gamma - E_{\text{binding}}$ where : $E_{\text{binding}} = E_K$ or E_L or $E_M \dots$

At low energy ($E_\gamma / m_e \ll 1$), but if $E_\gamma \gg E_K$:

$$\sigma^K_{photo} = \left(\frac{32}{\epsilon^7}\right)^{\frac{1}{2}} \alpha^4 \, Z^5 \, \sigma^e_{Th} \quad \text{(per atom)}$$

$\alpha = 1/137$ Fine structure constant

Thomson scattering cross section

$\epsilon = E_\gamma / m_e$

$$\sigma^e_{Th} = \frac{8}{3} \pi r_e^2 \text{ with } r_e = \frac{\alpha}{m_e} \text{ classical electron radius}$$

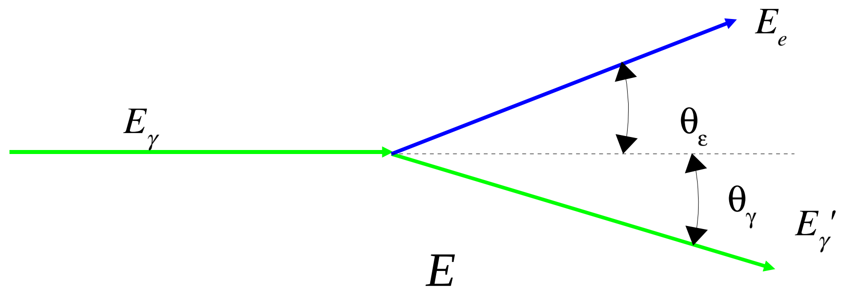
photoelectric cross section strongly increases as Z^5 and decreases as $1/E_\gamma^{3.5}$

At high energy ($E_\gamma / m_e \gg 1$) :

$$\sigma^K_{photo} = 4 \pi r_e^2 Z^5 \frac{\alpha^4}{\epsilon}$$

At low energy ($E_\gamma < 100$ keV) , the photoelectric effect dominates the total photon cross section

Compton scattering : Elastic scattering of a photon off an atomic electron considered as being free (if $E_\gamma > E_{binding}$)



$$\frac{E_\gamma'}{E_\gamma} = \frac{1}{1+\epsilon(1-\cos\theta_\gamma)} \quad with \;\; \epsilon = \frac{E_\gamma}{m_e}$$

$$cotg\,\theta_e = (1+\epsilon)\; tg\left(\frac{\theta_\gamma}{2}\right)$$

$$\frac{E_\gamma^{'\,min}}{E_\gamma} = \frac{1}{1+2\epsilon}$$

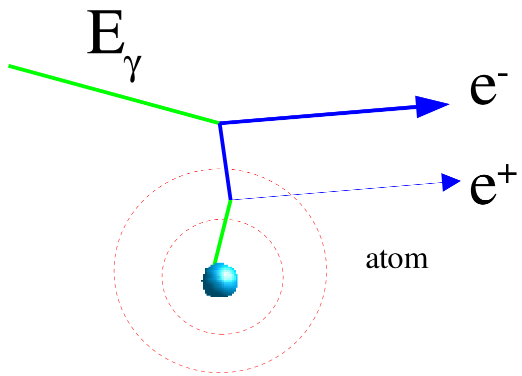
Klein-Nishina cross section :

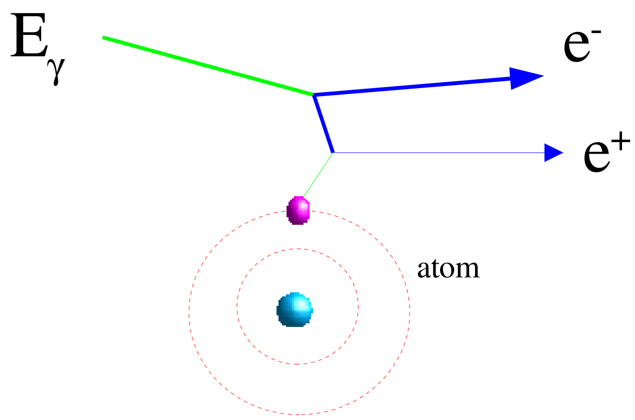
$$\sigma_c^e = 2\pi r_e^2 \left(\left(\frac{1+\epsilon}{\epsilon^2}\right) \left\{ \frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon}\ln(1+2\epsilon) \right\} + \frac{1}{2\epsilon}\ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2} \right) \;\; (\,per\ electron\,)$$

cross-section per atom :

$$\sigma_c^{atom.} = Z\ \sigma_c^e$$

e^+e^- pair production :





$E_\gamma \geqslant 2 m_e$ $\qquad$ $E_\gamma \geqslant 4 m_e$

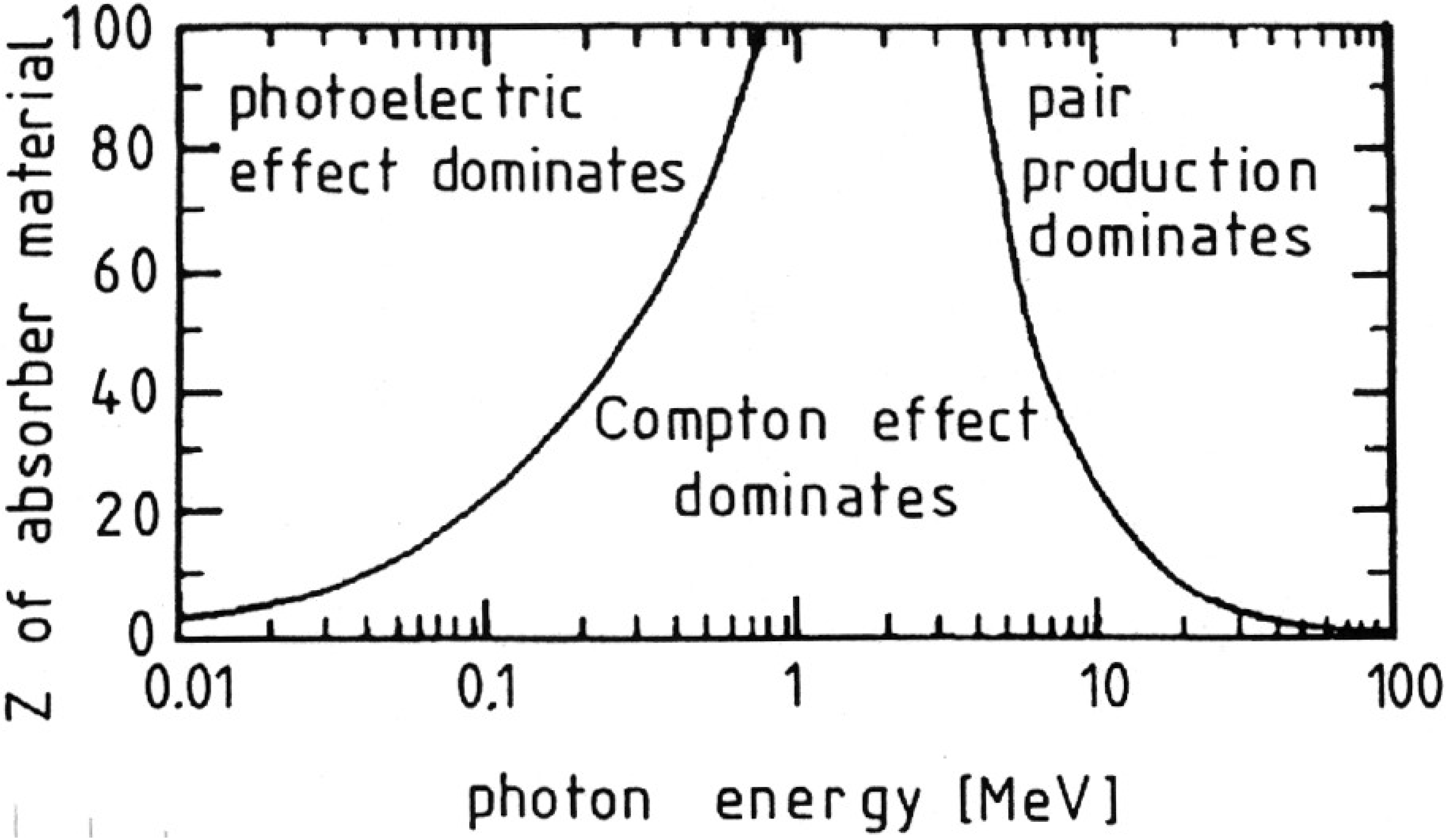
$$1 \ll \epsilon < \frac{1}{\alpha Z^{1/3}} \quad \Rightarrow \quad \sigma_{pair}^{atom.} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln(2\epsilon) - \frac{109}{54} \right)$$

$$\epsilon \gg \frac{1}{\alpha Z^{1/3}} \quad \Rightarrow \quad \sigma_{pair}^{atom.} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{54} \right)$$

In this high energy regime : $\sigma_{pair}^{atom.} \simeq \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$

where X_0 (g/cm^2) is the radiation length

Pair production is the leading effect at high energy


Z of absorber material
100
80
60
40
20
0
0.01
0.1
1
10
100
photon energy [MeV]
photoelectric effect dominates
Compton effect dominates
pair production dominates

Total absorption cross section :

In Compton scattering, photons are not totally absorbed

Let us define a Compton energy scattering cross section :

$$\sigma_{cs}^{atom.} = \frac{E'_\gamma}{E_\gamma}\,\sigma_c^{atom.}$$

And a Compton absorption cross section :

$$\sigma_{ca}^{atom.} = \sigma_c^{atom.} - \sigma_{cs}^{atom.}$$

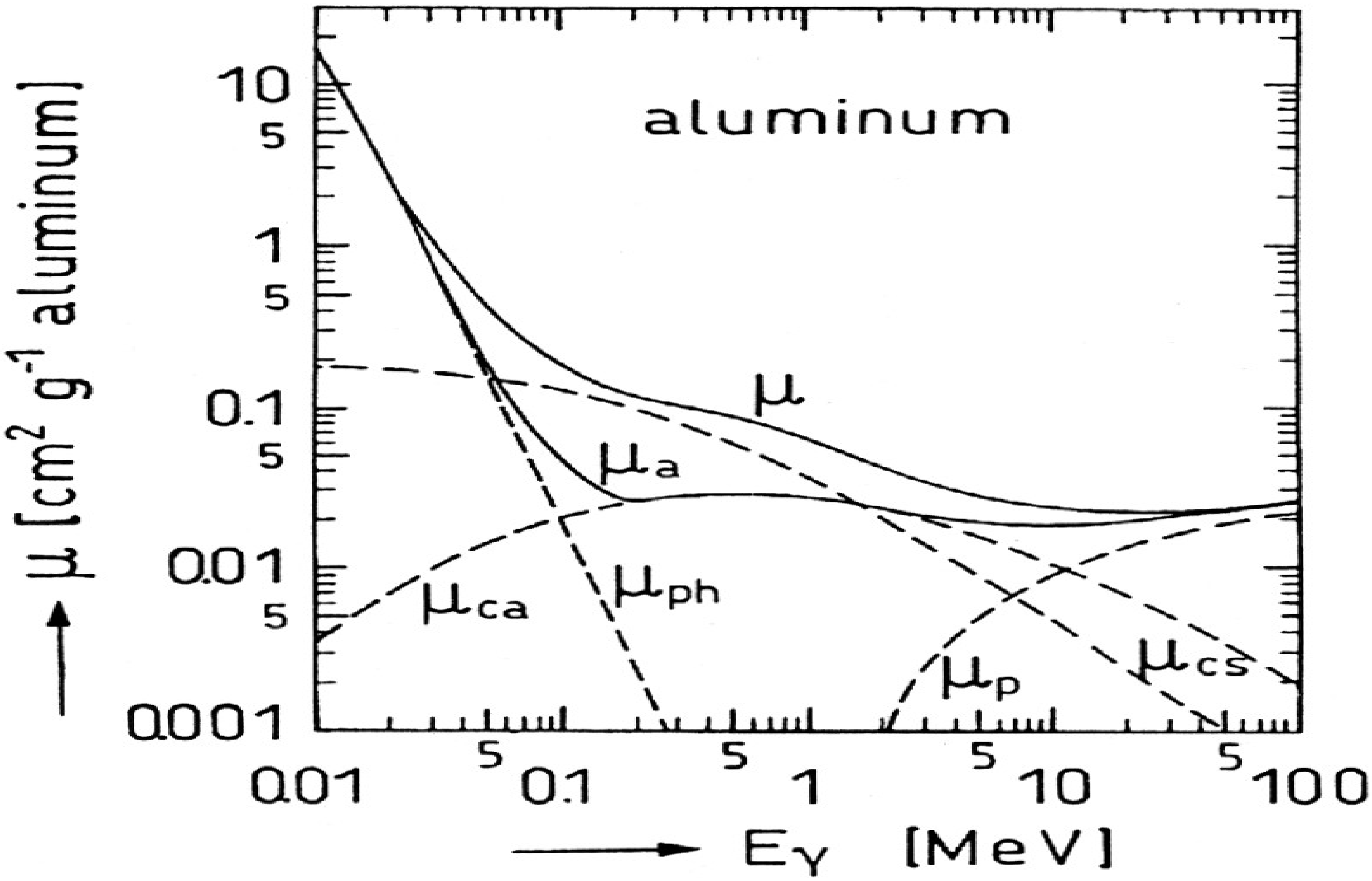
Massic coefficients in cm²/g :

$$\mu_{cs} = \frac{N}{A}\,\sigma_{cs}^{atom.} \quad ; \quad \mu_{ca} = \frac{N}{A}\,\sigma_{ca}^{atom.} \quad ; \quad \mu_c = \mu_{cs} + \mu_{ca}$$

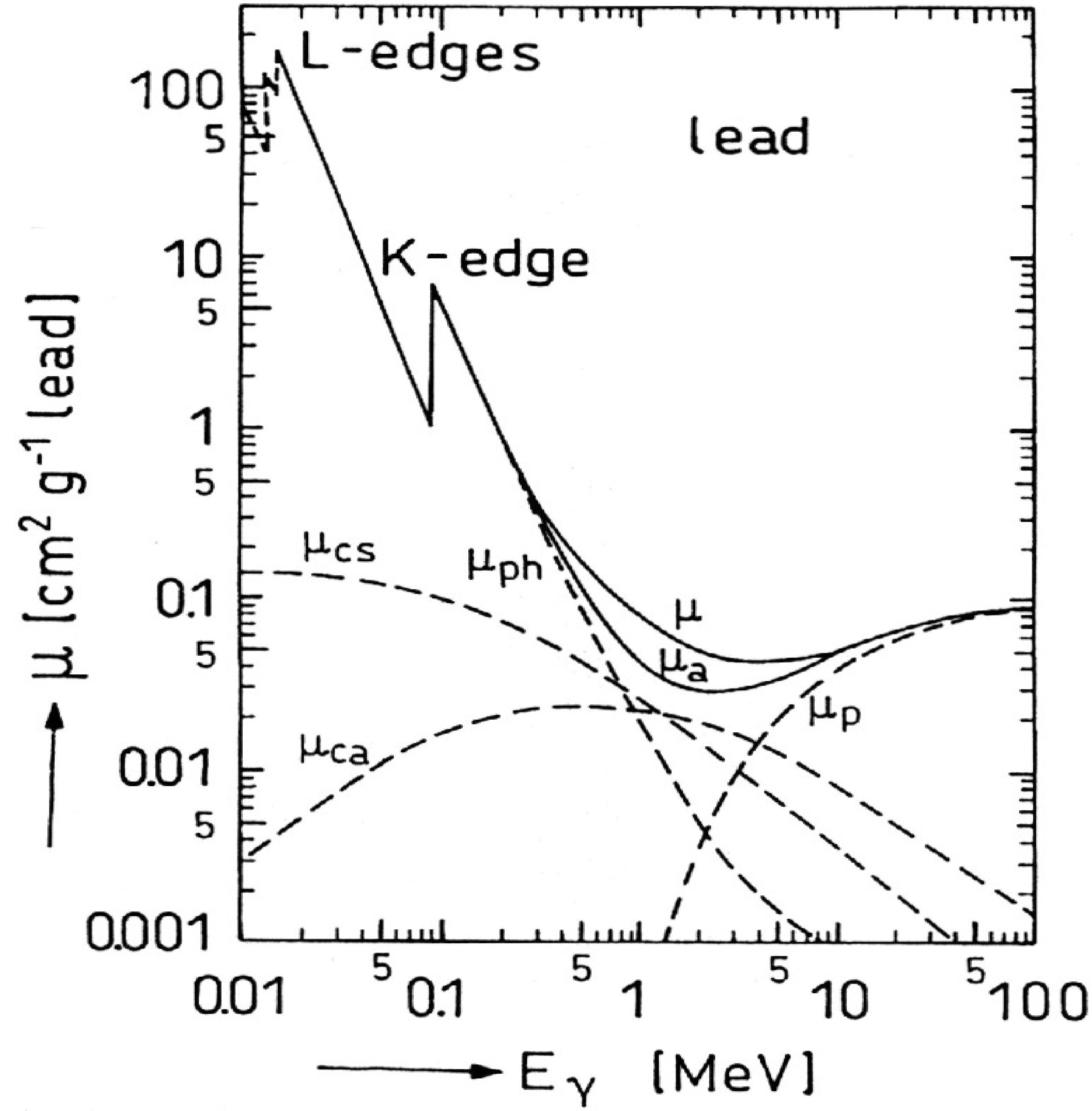
$$\mu_p = \frac{N}{A}\,\sigma_{pair}^{atom.} \quad ; \quad \mu_{ph} = \frac{N}{A}\,\sigma_{photo}$$

$\mu_a = \mu_{ph} + \mu_p + \mu_{ca}$ total massic absorption coefficient

$\mu = \mu_{ph} + \mu_p + \mu_c$ total massic attenuation coefficient



aluminum
10
5
1
5
0.1
5
0.01
5
0.001
μ [cm² g⁻¹ aluminum]
μ
μa
μph
μca
μp
μcs
0.01
5
0.1
5
1
5
10
5
100
Eγ [MeV]



L-edges
lead
K-edge
μ_{cs}
μ_{ph}
μ
μ_a
μ_p
μ_{ca}
100
5
10
5
1
5
0.1
5
0.01
5
0.001
0.01
5
0.1
5
1
5
10
5
100
μ [cm^2 g^{-1} lead]
E_γ [MeV]

Interactions of neutrons with matter :

Neutrons are neutral particles which only interact with nuclei.

Neutrons can be absorbed or scattered by nuclei .

With respect to their energy, neutrons can be categorized as follow :

- thermal neutrons : in thermal equilibrium with matter , $\langle E_n \rangle = 3/2\ K\ T$

  $\langle E_n \rangle = 0.038$ eV for $T = 300$ K   K being the Boltzmann constant

  E_0 (most probable energy) $= K\ T = 0.025$ eV for $T = 300$ K
- ultra-cold neutrons $E < 2\ 10^{-7}$ eV   - very cold neutrons $2\ 10^{-7}$ eV $< E < 50\ 10^{-6}$ eV
- cold neutrons $50\ 10^{-6}$ eV $< E < E_0 = 0.025$ eV
- slow neutrons 0.025 eV $< E <$ 0.5 eV
- epithermal neutrons : 0.5 eV $< E <$ 1 keV
- intermediate energy neutrons : 1 keV $< E <$ 0.5 MeV
- fast neutrons : 0.5 MeV $< E <$ 50 MeV
- relativistic neutrons : 50 MeV $< E$

All neutrons may undergo elastic scattering and radiative capture (emission of photons).

Elastic scattering :

It is mostly used to slow down neutrons , e.g. in a nuclear reactor.

^{1}H , ^{2}H and ^{12}C are the *best* and preferred moderator nuclei .

Up to 10 MeV and for some target nuclei (like H), the energy spectrum of elastically scattered neutrons is approximately flat :

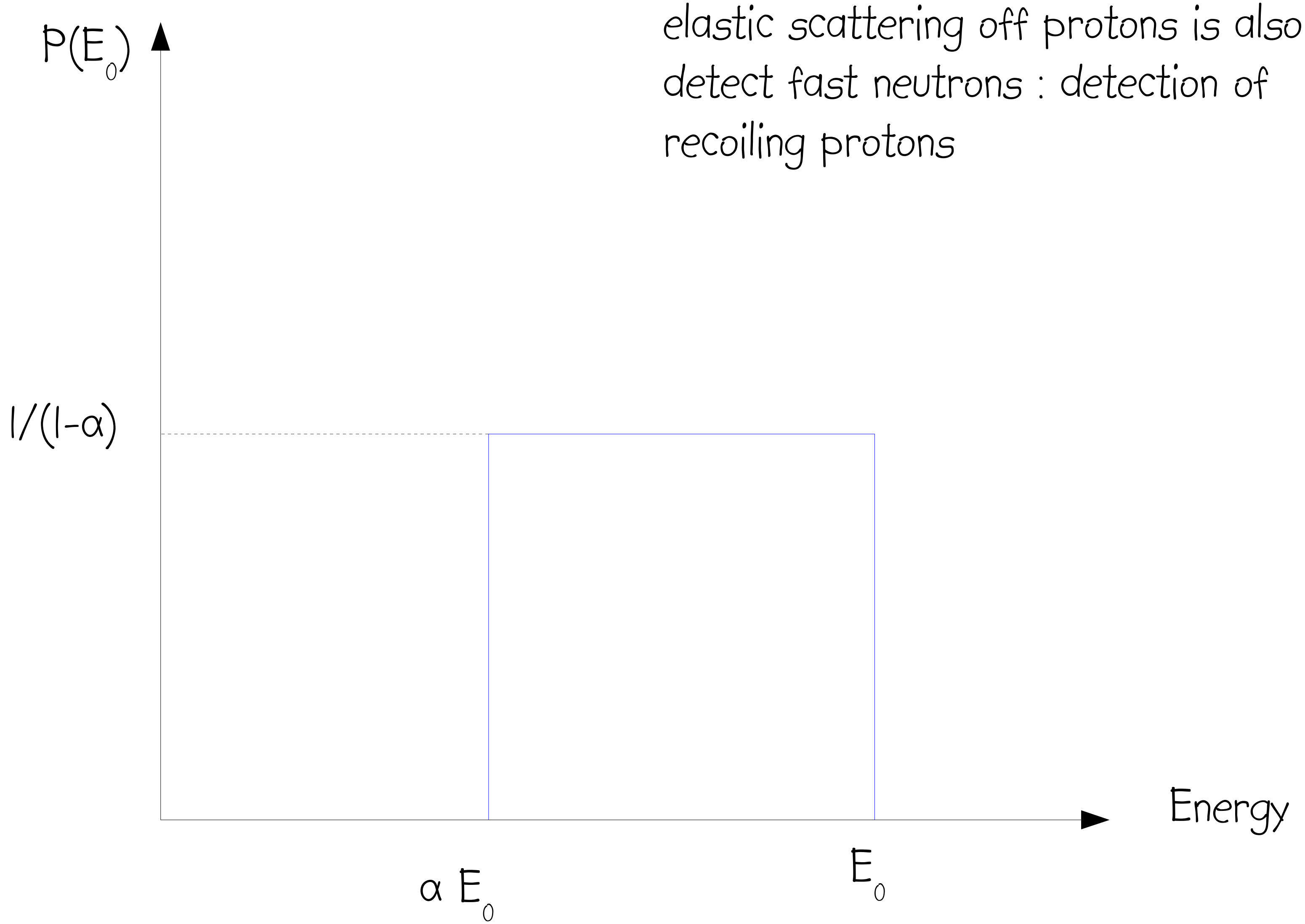
The probability of a neutron of mass m_n and incident energy E_0 to be found - after elastic scattering off a nucleus of mass m - in an energy interval dE is :

$$P(E_0)\,dE = \frac{dE}{(1-\alpha)E_0} \quad \text{where} \quad \alpha = \frac{(A-1)^2}{(A+1)^2} \leqslant 1$$

The scattered neutron energy follows :

$$\alpha E_0 \leqslant E \leqslant E_0$$

A is the mass number of the target nucleus elastic scattering off protons is also used to detect fast neutrons : detection of recoiling protons



Energy spectrum of elastically-scattered neutrons

(if the incident neutron energy is less than 10 MeV and the scattering process is approximately isotropic in the center-of-mass-coordinate system)

(n , γ) radiative capture : As neutrons are neutral particles, radiative capture may happen at very low energies (no Coulomb interaction effects)

example : Production of ^{239}Pu in a reactor

$$n + {}^{238}_{92}U \rightarrow {}^{239}_{92}U^* \rightarrow {}^{239}_{92}U + \gamma$$

$${}^{239}_{92}U \rightarrow {}^{239}_{93}Np + e^- + \bar{\nu}_e \rightarrow {}^{239}_{94}Pu + e^- + \bar{\nu}_e$$

(n , γ) radiative capture is used to produce artificial radioisotopes in nuclear reactors

It may also be used to detect neutrons and measure neutron fluences (time integrated fluxes).

In general, the capture cross section increases as the inverse of the neutron velocity : the slower the neutrons, the bigger the cross section (Gamow Law). This general behavior may be affected by capture resonances.

(n , γ n′) neutron inelastic scattering : neutron energy less than a few tens of MeV

gamma decay of nucleus

$$n+{}^{A}_{Z}X \;\rightarrow\; {}^{A+1}_{Z}X^{*} \;\rightarrow\; {}^{A}_{Z}X^{*}+n' \;\rightarrow\; {}^{A}_{Z}X+\gamma+n'$$

decay into neutron + excited nucleus

neutron capture

formation of compound nucleus

(n , p) and (n , α) reactions : A few of these reactions are exoenergetic (produce energy)

The neutron is first captured to produce a compound nucleus which then decays into several products.

$$n + {}^{3}He \rightarrow p + {}^{3}H + 760\ \text{keV} \qquad \sigma = 5400 \text{ barns}$$

$$n + {}^{14}N \rightarrow p + {}^{14}C + 630\ \text{keV} \qquad \sigma = 1.75 \text{ barns}$$

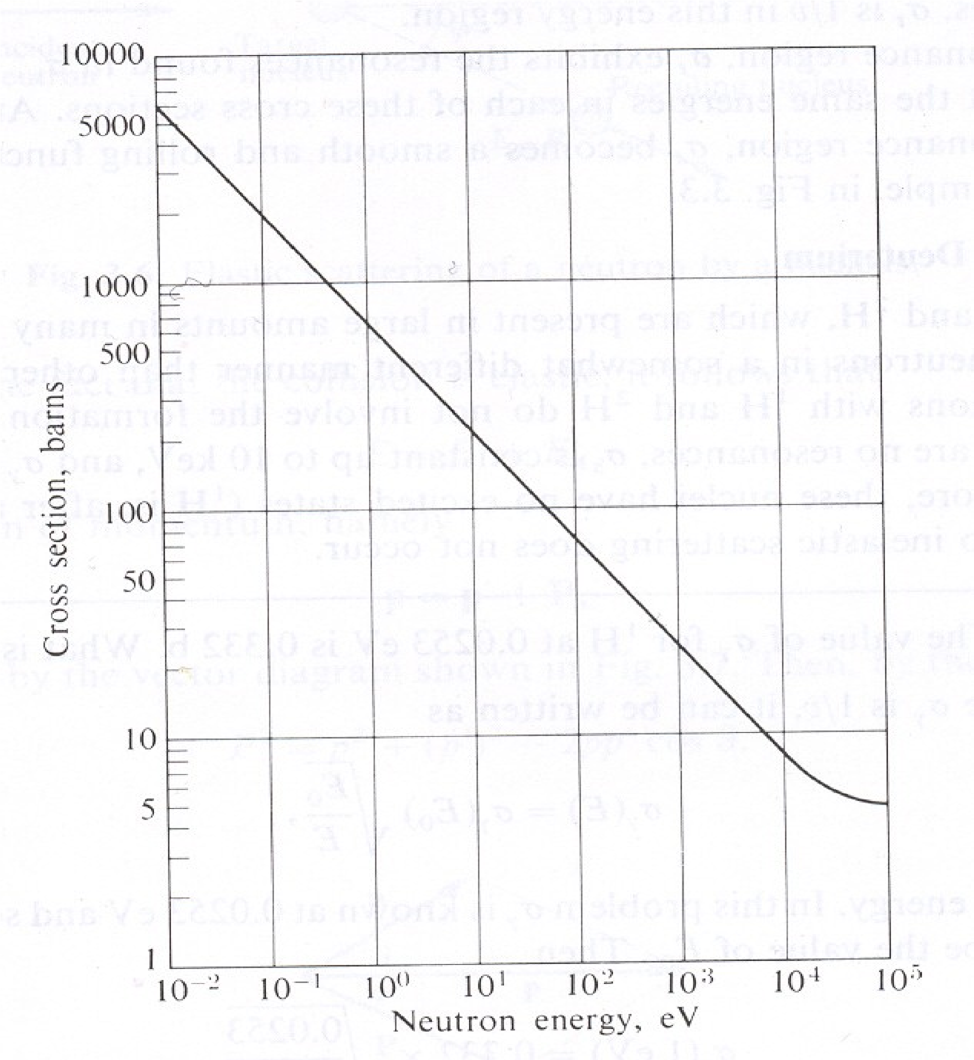
$$n + {}^{35}Cl \rightarrow p + {}^{35}S + 620\ \text{keV} \qquad \sigma = 0.8 \text{ barn}$$

$$n + {}^{6}Li \rightarrow \alpha + {}^{3}H + 4782\ \text{keV} \qquad \sigma = 940 \text{ barns}$$

$$n + {}^{10}B \rightarrow \alpha + {}^{7}Li + 2791\ \text{keV} \qquad \sigma = 4000 \text{ barns}$$

These reactions (in particular on ${}^{3}He$, ${}^{6}Li$, ${}^{10}B$) are used to detect low energy neutrons.

In general, the capture cross section increases as the inverse of the neutron velocity : the slower the neutrons, the bigger the cross section (Gamow Law). This general behavior may be affected by capture resonances.



Cross section of $^{10}B(n,alpha)^{7}Li$

neutron-induced fission :

fission fragments

$$n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X^{*} \rightarrow N + M + \text{a few neutrons} + 200\,MeV$$

For some odd neutron number nuclei (odd N), fission may occur at all neutron energies and in particular at very low energy . This is the case of :

^{233}U , ^{235}U , ^{239}Pu , ^{241}Pu

For other nuclei, fission only takes place above a neutron energy threshold, e.g.

^{238}U : 0.9 MeV , ^{232}Th : 1.3 MeV

Apart from its well-known application for massive energy production, fission may be used to detect neutrons.

Neutron cross sections :

microscopic cross section

Macroscopic cross section is defined as : $\Sigma = n\,\sigma$

Σ [cm^{-1}]

number of nuclei per unit volume

$$\Sigma_{tot} = \Sigma_{\text{elastic scattering}} + \Sigma_{\text{absorption}} + \Sigma_{\text{inelastic scattering}}$$

As in the case of gammas, we can use an attenuation law :

initial neutron flux

$$I(x) = I_0 e^{-\Sigma_{tot} x}$$

distance traversed by neutrons

$\lambda = \dfrac{1}{\Sigma_{tot}}$ being the neutron mean free path in the considered medium

Deposited energy :

Generally speaking, the energy loss is never equal to the deposited energy as the radiated photons or the secondary particles may escape the medium.

Deposited energy is what generates the signal in a particle detector.

Deposited energy is subjected to large stochastic fluctuations. Remember : Stopping power is the mean energy loss.
If the medium is thin and the number of interactions is small, the deposited energy distribution is asymmetric : it is sometimes called a Landau distribution.
If the medium is thick or the number of interactions is large, the deposited energy distribution tends to a Gaussian.

There are no simple and exact analytical formulae to compute deposited energy.

Nowadays, to estimate the energy deposited in a detector or more generally in a medium we use a Monté-Carlo program which simulates the propagation of the particle through matter : e.g. Geant4 creation of electron-ion pairs :

When the measured signal is a current or a charge liberated through ionizing interactions, it is useful to compute the mean number of created electron-ion pairs :

$$n^{\text{e - ion}} = \frac{\Delta E_{deposited}}{W}$$

where : W is the required mean energy to produce an e-ion pair

W > I (mean excitation and ionization potential)

In most gazes, W ~ 30 eV.

In semiconductor detectors (Ge, Si), W is much lower : e.g. W=3.6 eV for Si and W=2.85 eV for Ge

To learn more :

- Principles of Radiation Interaction in Matter and Detection, C. Leroy and P.G. Rancoita World Scientific

- Nuclei and particles, Émilio Segré, W.A. Benjamin

- Stopping powers and ranges for protons and alpha particles (ICRU Report 49,1993) Library of congress US-Cataloging-in-Publication Data

- Particle detectors, Claus Grupen, Cambridge monographs on particle physics

- Detectors for Particle radiation, Konrad Kleinknecht, Cambridge University Press

- Radiation detection and measurement, G.F. Knoll, J. Wiley & Sons

- Single Particle Detection and Measurement, R. Gilmore, Taylor & Francis

- Radiation detectors, C.F.G. Delaney and E.C. Finch , Oxford Science Publications

- High-Energy Particles, Bruno Rossi, Prentice-Hall

- Introduction to nuclear engineering, John R. Lamarsh, Addison-Wesley Publishing