Interaction of particles with matter (lecture 1)

A brief review of a few typical situations is going to greatly simplify the subject.

Mean free path of a particle, i.e. average distance travelled between two consecutive collisions in matter :

$$\lambda = \frac{1}{\sigma n}$$

where :

σ total interaction cross-section of the particle

n number of scattering centers per unit volume

example : $n = \frac{\rho N_A}{M}$ for a monoatomic element of molar mass M and specific mass ρ .

N_A Avogadro number

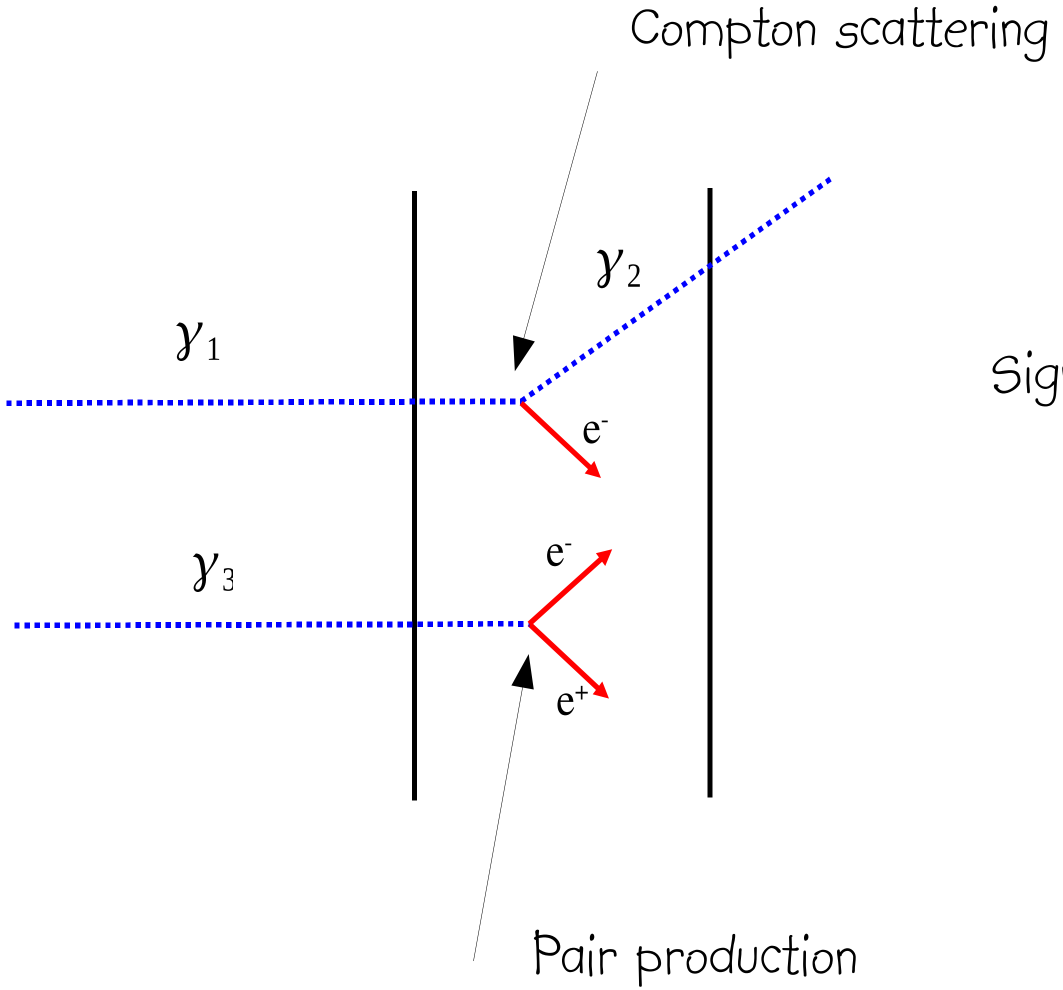
Electromagnetic interaction : $\lambda \leqslant 1 \mu \mathrm{m}$ (charged particles)

Strong interaction : $\lambda \geqslant 1 \mathrm{cm}$ (neutrons ....)

Weak interaction : $\lambda \geqslant 10^{15} \mathrm{m} \simeq 0{,}1$ light year (neutrinos)

A practical signal (>100 interactions or hits) can only come from electromagnetic interaction

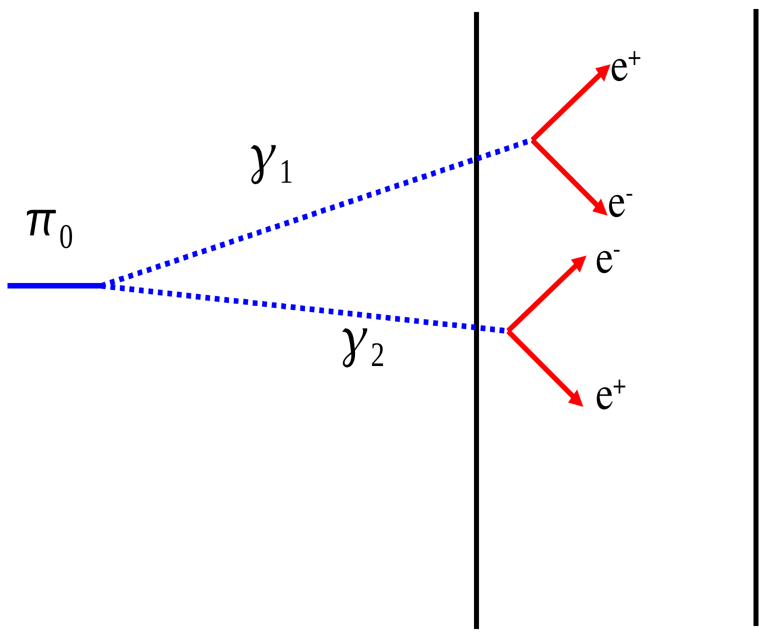
Particle detection proceeds in two steps : 1) primary interaction 2) charged particle interaction producing the signals typical examples : photon detection



Signal is induced by electrons

neutral pion detection :

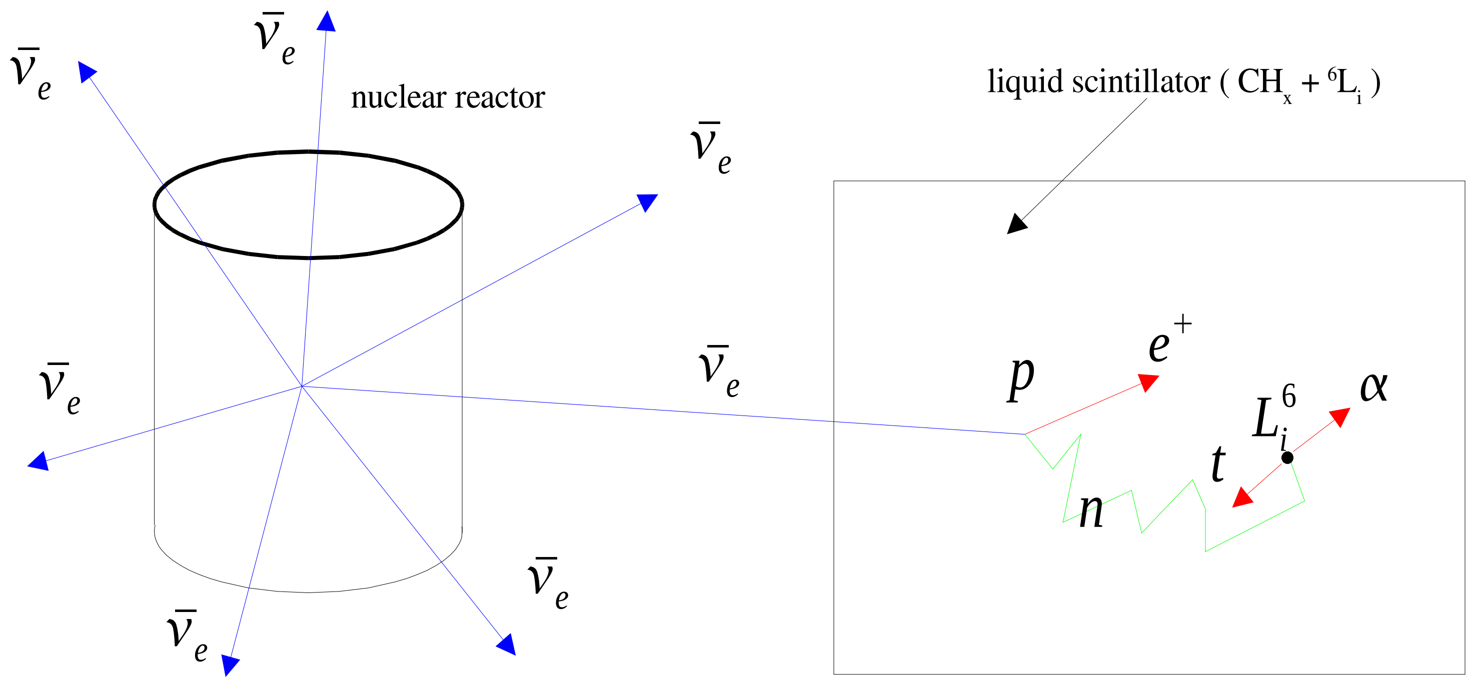
A π^0 decays into two photons with a mean lifetime of $8.5\ 10^{-17}$ s.



neutrino detection :

A 2800 MW nuclear power station produces 130 MW of neutrinos !

A detector of 1 m^3 located 20 m away from the reactor core can detect 100 neutrinos/h.



$\epsilon_\nu \sim 0$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$n_{th} + L_i^6 \rightarrow \alpha + t + 4{,}8\ MeV$$

Charged particles produce light in the target scintillator

Interaction of charged particles with matter

For heavy particles ionization and excitation are the dominant processes producing energy loss.

Particle P of Z charge state

Excitation : $P^{(Z)} + \mathrm{atom} \rightarrow \mathrm{atom}^{*} + P^{(Z)}$ followed by : $\mathrm{atom}^{*} \rightarrow \mathrm{atom} + \gamma$

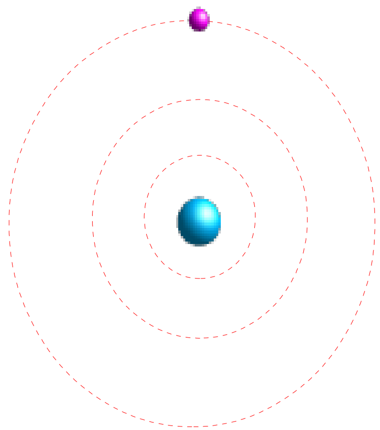
Ionization : $P^{(Z)} + \mathrm{atom} \rightarrow \mathrm{atom}^{+} + e^{-} + P^{(Z)}$

Ionization + excitation : $P^{(Z)} + \mathrm{atom} \rightarrow \mathrm{atom}^{*+} + e^{-} + P^{(Z)}$

T_e^{max} , Maximal kinetic energy transferred to an ionized electron :

m_0 $\vec{v}$

m_e



atom

Z

$$\beta=\frac{v}{c}$$

$$\vec{p}=\gamma m_0 \vec{v}$$

$$E=(p^2+m_0^2)^{-1/2}$$

$$\gamma=(1-\beta^2)^{-1/2}$$

$$\beta=(1-\gamma^{-2})^{1/2}$$

Hypothesis : $V > \langle v_e \rangle = Z \alpha c$, speed of deepest atomic orbit electrons where α is the fine structure constant : $\alpha = 1/137$.

One may show (exercise) that : $T_e^{max}=E_e^{max}-m_e=\frac{2 m_e \beta^2 \gamma^2}{(E_{CM}/m_0)^2}$ (In natural units , $c=\hbar=1$)

where : $E_{CM} = (m_0^2+m_e^2+2 m_e E)^{1/2}$ total energy in center-of-mass frame

Two cases :

$m_0 \gg m_e$, i.e. the incoming particle is not an electron and if its energy is not too big

$$(E_{CM}/m_0)^2 = \left(\frac{m_0^2}{m_0^2} + \frac{m_e^2}{m_0^2} + \frac{2 m_e E}{m_0^2}\right) \simeq 1 \qquad \text{with} \qquad E = \gamma m_0$$

$\frac{2\gamma m_e}{m_0} \ll 1$ proton $E_p < 50$ GeV , muon $E_\mu < 500$ MeV (medium energy range)

then : $T_e^{max} = E_e^{max} - m_e = 2 m_e \beta^2 \gamma^2$

$m_0 = m_e$ the incoming particle is an electron

$T_e^{max} = (E - m_e)$ due to undistinguishibility of electrons, max transferable energy = T^{max} / 2

If the incoming particle is not an electron then in practice $m_0 \gg m_e$.

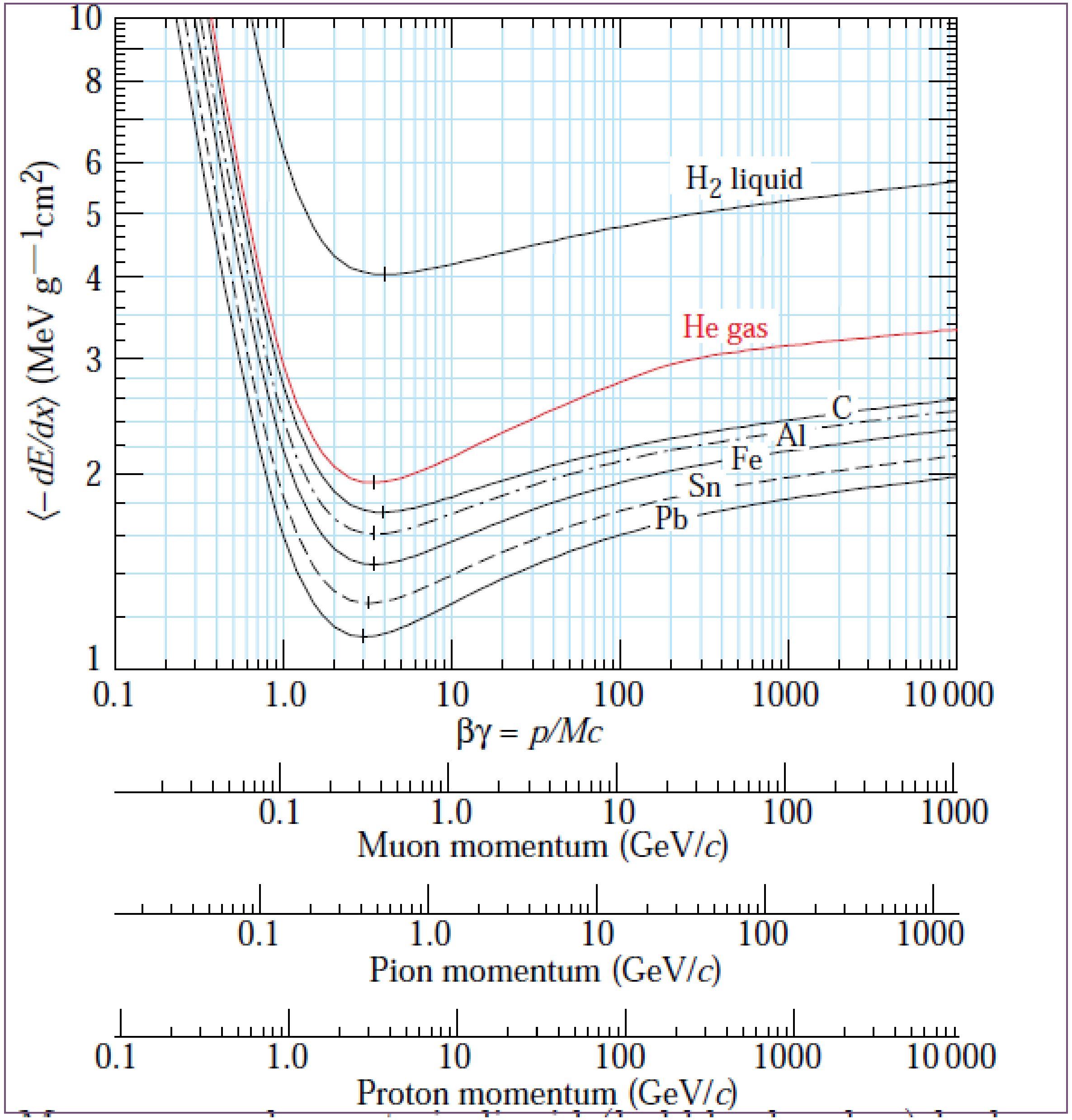
Stopping power of heavy particles by excitation and ionization in matter.
Average energy loss by a charged particle (other than an electron) in matter.

Bethe and Bloch formula

(see Nuclei and particles, Émilio Segré, W.A. Benjamin ; Principles of Radiation Interaction in Matter and Detection, C. Leroy and P.G. Rancoita, World Scientific ; Introduction to experimental particle physics, R. Fernow)

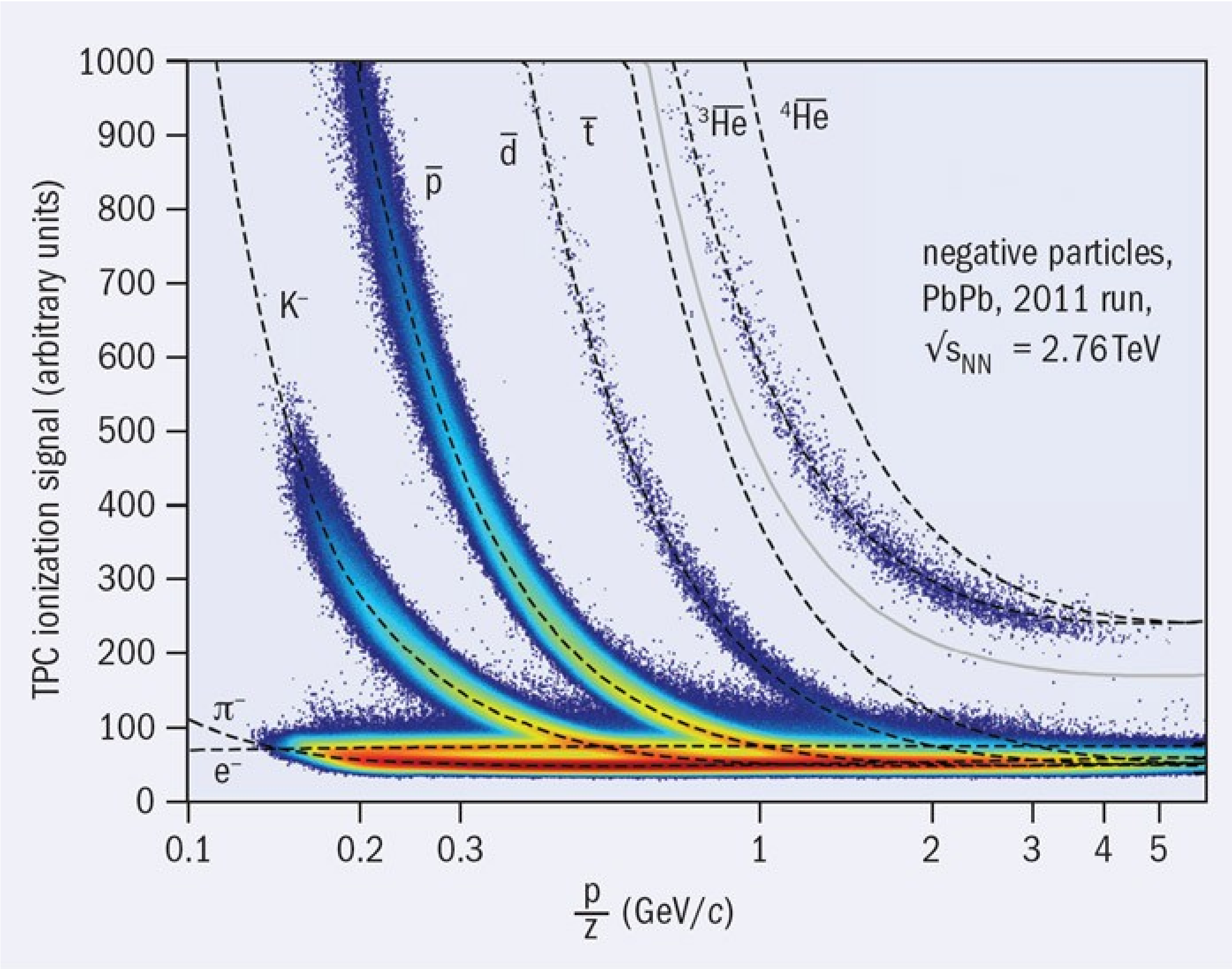
$$-\left(\frac{dE}{dx}\right)\left[\frac{\text{MeV}}{\text{g/cm}^2}\right] = \frac{0.3071}{A(\text{g mol}^{-1})}\frac{z^2\ Z}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{2m_e\beta^2\gamma^2T_e^{max}}{I^2}\right)-\beta^2-\frac{\delta(\gamma\beta)}{2}-\frac{C_e}{Z}\right)$$

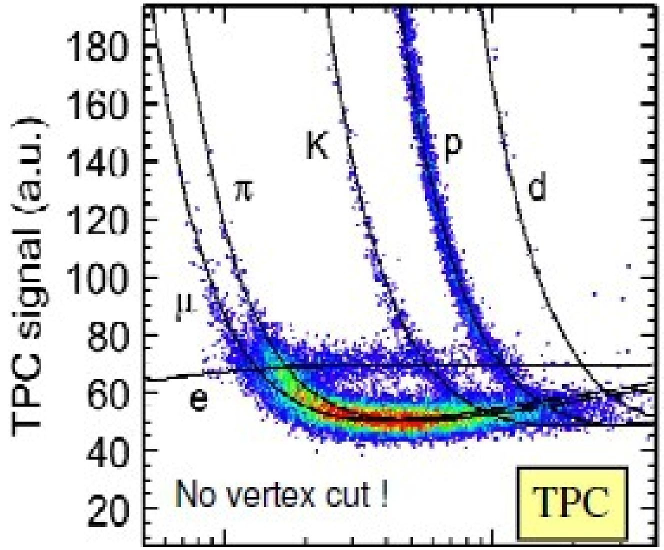
- $\frac{dE}{dx}$: Stopping power or mean specific energy loss
- dx: Surface mass density of medium $dx = \rho\, dl$ (or mass thickness of medium)
- A: Atomic mass of medium
- z: charge of incoming particle
- Z: Z of medium
- I: mean excitation energy
- $\delta(\gamma\beta)$: density effect correction at high energy
- C_e: Atomic shell correction at low energy (not covered in this lecture, see Leroy & Rancoita)



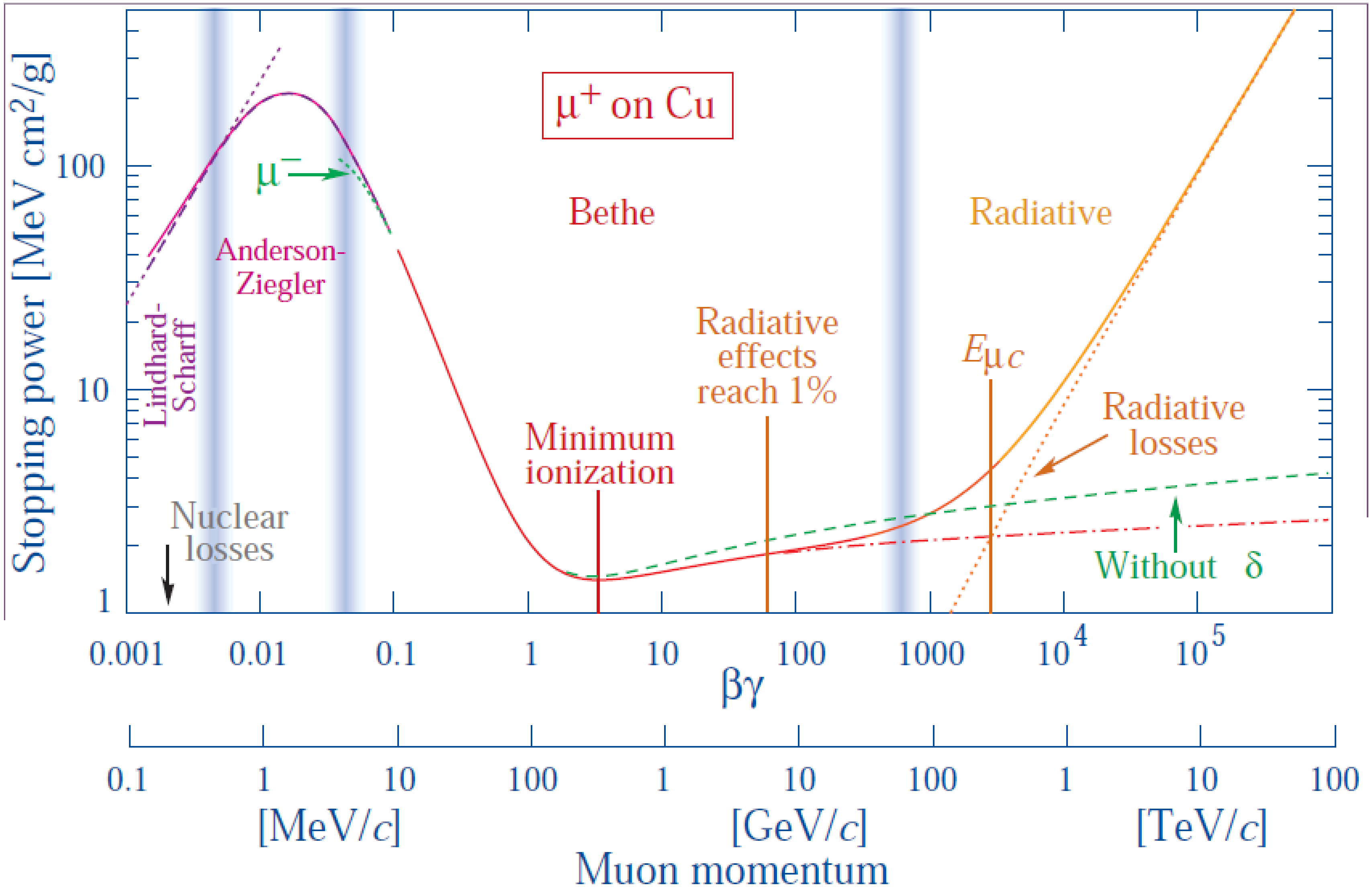
show that :

$$\beta^2 = \frac{(\beta\gamma)^2}{1+(\beta\gamma)^2}$$





Particle identification in Alice TPC



μ+ on Cu
Stopping power [MeV cm²/g]
100
10
1
μ−
Bethe
Radiative
Anderson-
Ziegler
Lindhard-
Scharff
Radiative
effects
reach 1%
Eμc
Radiative
losses
Minimum
ionization
Nuclear
losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10⁴
10⁵
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

few remarks :

- for βγ < 1 : $-\frac{dE}{dx} \sim \beta^{-5/3}$ non relativistic particles

- for βγ ~ 3-4 : $-\frac{dE}{dx}$ is minimal over a large energy plateau . A particle in this state is called a minimum ionizing particle (MIP)

In media composed of light elements : $-\frac{dE}{dx}^{MIP} \simeq 2 \ \frac{\text{MeV}}{\text{g cm}^{-2}}$

- for βγ > 4 : relativistic increase of $-\frac{dE}{dx}$ as $\ln(\gamma)$ wich is tempered by -δ/2 correction.

- I : mean excitation and ionization energy , I = 15 eV for atomic H and 19.2 eV for H_2

I = 41.8 for He

I = 15 $Z^{0.9}$ eV for Z > 2

At medium energy : $\frac{2\gamma m_e}{m_0} \ll 1$ $T_e^{max} = 2 m_e \beta^2 \gamma^2$

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right] = \frac{0.3071}{A(g)} \cdot \frac{z^2 Z}{\beta^2}\left[\ln\left(\frac{2 m_e \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2} - \frac{C_e}{Z}\right]$$

Density effect correction

When energy increases, stopping power decreases to a minimum ($1/\beta^2$ dependance) and then starts rising again due to logarithmic term. In fact, the max. transverse electric field increases as γ but its influence is screened by nearby atoms beyond a distance of 70 $(A(g)/\rho(g)Z)^{1/2}$ Å (shown by Bohr). This density effect tempers the relativistic rise.

Studies have be carried-out by Sernheimer, Peierls, Berger & Seltzer (see Leroy & Rancoita).

The density correction effect term, δ is given by :

for $\beta\gamma < 10^{S_0}$: $\delta = \delta_0 \left(\frac{\beta\gamma}{10^{S_0}}\right)^2$

for $10^{S_0} < \beta\gamma < 10^{S_1}$: $\delta = 2\ln(\beta\gamma) + C + a\left[\frac{1}{\ln(10)}\ln\left(\frac{10^{S_1}}{\beta\gamma}\right)\right]^{md}$

for $\beta\gamma > 10^{S_1}$: $\delta = 2\ln(\beta\gamma) + C$

where : $C = -2\ln\left(\frac{I}{h\nu_p}\right) - 1$

with $\nu_p = \sqrt{\frac{n r_e c^2}{\pi}}$ in which n is the density of electrons and r_e the classical radius of e^- : r_e = 2.82 fm

$$h\nu_p \simeq 28.7\sqrt{\frac{\rho(g/cm^3)}{A(g)}Z}\ eV$$

show that at very high energy : $\beta\gamma > 10^{S_1}$

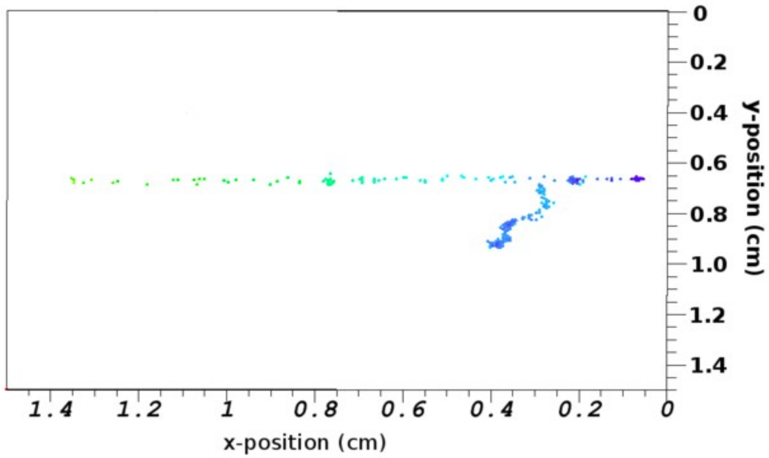
$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right] = 0.3071\,\frac{z^2 Z}{2.A(g)}\left[\ln\left(\frac{2m_e T_e^{max}}{(h\nu_p)^2}\right) - 1\right]$$

Table 2.1 Values of Z, Z/A, I, ρ in units of g/cm^3, $h\nu_p$ and density-effect parameters S_0, S_1, a, md, and δ_0 for elemental substances.

El.	Z	Z/A	I eV	ρ	$h\nu_p$ eV	S_0	S_1	a	md	δ_0
He	2	0.500	41.8	1.66 10^{-4}	0.26	2.202	3.612	0.134	5.835	0.00
Li	3	0.432	40.0	0.53	13.84	0.130	1.640	0.951	2.500	0.14
O	8	0.500	95.0	1.33 10^{-3}	0.74	1.754	4.321	0.118	3.291	0.00
Ne	10	0.496	137.0	8.36 10^{-4}	0.59	2.074	4.642	0.081	3.577	0.00
Al	13	0.482	166.0	2.70	32.86	0.171	3.013	0.080	3.635	0.12
Si	14	0.498	173.0	2.33	31.06	0.201	2.872	0.149	3.255	0.14
Ar	18	0.451	188.0	1.66 10^{-3}	0.79	1.764	4.486	0.197	2.962	0.00
Fe	26	0.466	286.0	7.87	55.17	-0.001	3.153	0.147	2.963	0.12
Cu	29	0.456	322.0	8.96	58.27	-0.025	3.279	0.143	2.904	0.08
Ge	32	0.441	350.0	5.32	44.14	0.338	3.610	0.072	3.331	0.14
Kr	36	0.430	352.0	3.48 10^{-3}	1.11	1.716	5.075	0.074	3.405	0.00
Ag	47	0.436	470.0	10.50	61.64	0.066	3.107	0.246	2.690	0.14
Xe	54	0.411	482.0	5.49 10^{-3}	1.37	1.563	4.737	0.233	2.741	0.0
Ta	73	0.403	718.0	16.65	74.69	0.212	3.481	0.178	2.762	0.14
W	74	0.403	727.0	19.30	80.32	0.217	3.496	0.155	2.845	0.14
Au	79	0.401	790.0	19.32	80.22	0.202	3.698	0.098	3.110	0.14
Pb	82	0.396	823.0	11.35	61.07	0.378	3.807	0.094	3.161	0.14
U	92	0.387	890.0	18.95	77.99	0.226	3.372	0.197	2.817	0.14

Data are from [Sternheimer, Berger and Seltzer (1984)]

Restricted energy loss



Knock-on electron (delta ray) generated by a 180 GeV muon as observed by the experiment GridPix at CERN SPS.

High energy transfers generate delta rays that may espace the detector if it is too thin. So average energy deposits are very often much smaler thant predicted by B&B.

If T_0 is the average maximal delta ray energy that can be absorbed in the detecting medium, a better estimate of the average deposited energy is given by :

$$-\left(\frac{dE}{dx}\right)\left[\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\right] = \frac{0.3071}{A(\mathrm{g\ mol^{-1}})}\frac{z^2\ Z}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{2m_e\beta^2\gamma^2T_0}{I^2}\right)-\frac{\beta^2}{2}\left(1+\frac{T_0}{2m_e\beta^2\gamma^2}\right)-\frac{\delta(\gamma\beta)}{2}-\frac{C_e}{Z}\right)$$

At extremely high energies, when $\beta\gamma > 10^{S_1}$, stopping power reaches a constant called Fermi plateau.

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=0.3071\frac{z^2 Z}{2.A(g)}\ln\left(\frac{2m_e T_0}{(h\nu_p)^2}\right)$$

Fermi plateau measured in silicon

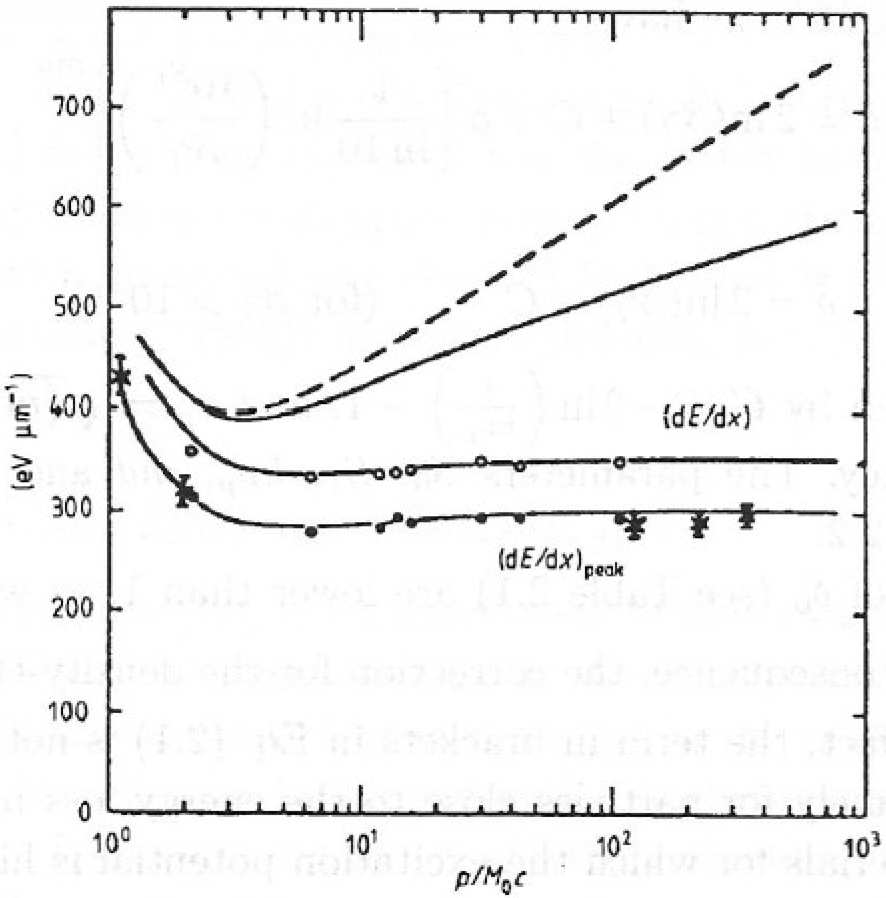


Fig. 2.5 Energy loss in silicon (in units of eV/μm) versus $\beta\gamma$ ($= p/M_0c$, where M_0 is the rest mass of the incoming particle) from [Rancoita (1984)]. From the top the first two curves are: the $-dE/dx$ without (broken curve) and with (full curve) the density-effect correction. The following second two curves are compared to experimental data for detector thicknesses of 300 ($\times$ from [Hancock, James, Movchet, Rancoita and Van Rossum (1983)]) and 900 μm ($\circ$ and $\bullet$ from [Esbensen et al. (1978)]): the restricted energy loss with the density-effect taken into account and the prediction of the most probable energy loss.

Delta rays (secondary electrons)

The differential probability to generate a delta ray of kinetic energy T is given by :

$$\frac{dw(T,E)}{dTdx} = 0.3071 \frac{z^2 Z}{2.A(g)\beta^2} \frac{F(T)}{T^2} \; MeV^{-1} cm^2 g^{-1}$$

F(T) is a spin-dependent factor.

$$F(T) = F_0(T) = (1 - \beta^2 \frac{T}{T_{max}})$$ for spin-0 particles

$$F(T) = F_{1/2}(T) = F_0(T) + \frac{1}{2} (\frac{T}{E})^2$$ for spin-1/2 particles

$$F(T) = F_1(T) = F_0(T)(1 + \frac{1}{3} \frac{T m_e}{m_0^2}) + \frac{1}{3} (\frac{T}{E})^2 (1 + \frac{1}{2} \frac{T m_e}{m_0^2})$$ for spin-1 particles

Delta rays (secondary electrons)

For $T \ll T_{max}$ and $T \ll m_0^2 / m_e$,

$$\frac{dw(T,E)}{dTdx} = 0.3071 \frac{z^2 Z}{2.A(g)\beta^2} \frac{1}{T^2} \; MeV^{-1} cm^2 g^{-1}$$

This allows to compute an approximate probability to generate a delta ray of kinetic energy greater than T_s in a thin absorber of mass thickness x :

$$w(Ts, E, x) \simeq 0.3071\, x \frac{z^2 Z}{2.A(g)\beta^2} \frac{1}{T_s}$$

show this expression

So far, only the average energy loss has been considered. But energy loss is subjected to large fluctuations that in thin absorbers results in asymmetric distributions. The subject is quite complex and has no general exact solutions, but a few approximate formulas help to estimate it.

For thin absorbers in which $\epsilon / T_{max} \ll 1$, where : $\epsilon = 0.3071\, x \dfrac{z^2 Z}{2.\, A(g)\beta^2}\ MeV$

where x is mass thickness of the absorber in g cm^{-2}.

The problem was first studied by Landau and then Vavilov. Their distribution functions are not analitic. A useful approximation of the Landau distribution is :

$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda})\right)$$

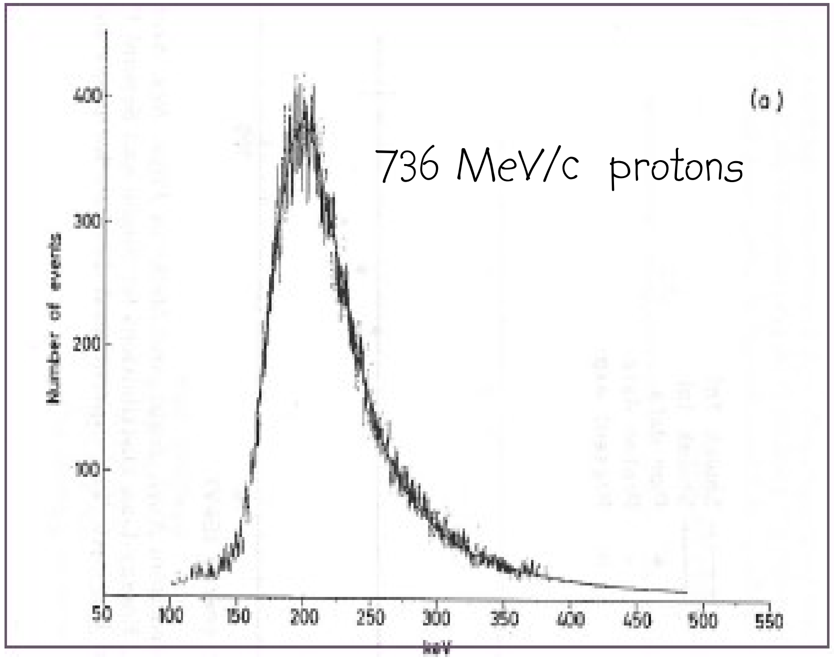
where : $\lambda = \dfrac{\Delta E - \Delta E_{MP}}{\epsilon}$ and ΔE is the energy loss, ΔE_{MP} is the most probable energy loss.

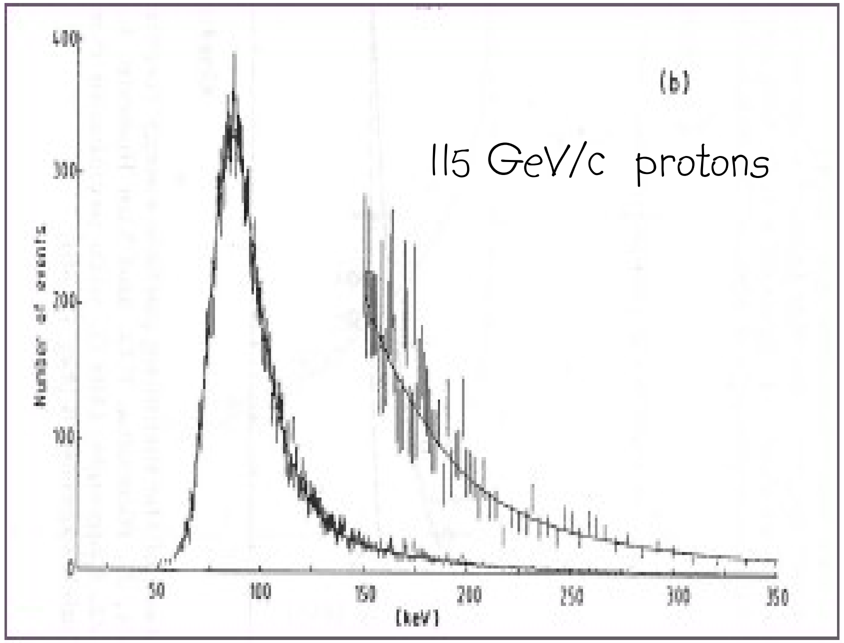
$$\Delta E_{MP} = \Delta E_{Bethe} + \epsilon \left(\beta^2 + \ln\left(\frac{\epsilon}{T_{max}}\right) + 0.194\right)\ MeV$$

Energy straggling

Still for thin absorbers, an improved generalized energy loss distribution that takes into account the distant collisions that are neglected in Landau's approach, can be obtained by convoluting a Landau distribution with a Gaussian distribution :

$$f(\Delta E, x)_I = \frac{1}{\sqrt{2\pi\sigma_I^2}} \int_{-\infty}^{+\infty} L(\Delta E - \Delta E', x) \exp\left(\frac{-\Delta E'}{2\sigma_I^2}\right) d(\Delta E')$$





Energy deposited by protons in silicon

Fig. 2.10 Curves (a) and (b) (adapted and republished with permission from Hancock, S., James, F., Movchet, J., Rancoita, P.G. and Van Rossum, L., *Phys. Rev. A* **28**, 615 (1983); Copyright (1983) by the American Physical Society) show the energy loss spectra at 0.736 and 115 GeV/c of incoming particle momentum. Continuous curves are the complete fit to experimental data, i.e., the Landau straggling function folded over the Gaussian distribution taking into account distant collisions.

Energy straggling

In thick absorbers in which $\epsilon / T_{max} \gg 1$, both the Landau and the Vavilov distributions tend to a Gaussian :

$$f(\Delta E, x) \simeq \frac{1}{\sqrt{2\pi T_{max}\epsilon(1-\frac{\beta^2}{2})}} \exp\left(-\frac{(\Delta E-\Delta E_{Bethe})^2}{2 T_{max}\epsilon(1-\frac{\beta^2}{2})}\right)$$

with :

$$\sigma \simeq \sqrt{\epsilon T_{max}(1-\frac{\beta^2}{2})}$$

Stopping power of electrons by ionization and excitation in matter.

Incoming and outgoing particles are identical.
Energy transfer is bigger .

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=\frac{0.3071}{A(g)}\cdot\frac{Z}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{T m_e\beta^2\gamma^2}{2I^2}\right)+\frac{1}{2\gamma^2}\left(1-(2\gamma-1)\ln(2)\right)+\frac{1}{16}\left(\frac{\gamma-1}{\gamma}\right)^2\right]$$

Kinetic energy of incoming electron : $T=(\gamma-1)m_e=E-m_e$

Stopping power of positrons by ionization and excitation in matter.

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=\frac{0.3071}{A(g)}\cdot\frac{Z}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{T m_e\beta^2\gamma^2}{2I^2}\right)-\frac{\beta^2}{24}\left(23+\frac{14}{\gamma+1}+\frac{10}{(\gamma+1)^2}+\frac{4}{(\gamma+1)^3}\right)\right]$$

When a positron comes to a rest it annihilates : $e^+ + e^- \rightarrow \gamma\,\gamma$ of 511 KeV each

A positron may also undergo an annihilation in flight according to the following cross section :

$$\sigma(Z,E)=\frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

Stopping power of a compound medium

$$\frac{dE}{dx} \approx \sum_i f_i \left.\frac{dE}{dx}\right|_i$$

$$f_i = \frac{m_i}{m} \quad , \quad \sum_i m_i = m$$ where f_i is the massic ratio of element i

$\left.\frac{dE}{dx}\right|_i$ is the stopping power of element i

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Interaction of particles with matter (lecture 2)

Bremsstrahlung : electromagnetic radiative energy loss

A decelerated or accelerated charged particle radiates photons.
The mean radiative energy loss is given by :

$$-\frac{dE}{dx}^{rad}\left(\frac{MeV}{g/cm^2}\right)=\frac{0.3071}{A(g)}\frac{\alpha}{\pi}Z^2 z^2\left(\frac{m_e}{m}\right)^2\frac{E}{m_e}\ln\left(\frac{183}{Z^{1/3}}\right)$$

fine structure constant = 1/137

medium atomic number

incoming particle energy

incoming particle mass

incoming particle charge state

The mean radiative energy loss of a particle of charge z and mass m is a function of the mean radiative energy loss of an electron :

$$\frac{dE}{dx}^{rad}(z,m)=\left(\frac{m_e}{m}\right)^2 z^2\ \frac{dE}{dx}^{rad}(e^-)$$

Electrons are much more sensitive to this effect.

For an electron and taking into account the Bremsstrahlung radiation induced by atomic electrons :

classical radius of electron : $r_e = \alpha / m_e$

$$-\frac{dE}{dx}^{rad}(e^-) = 4\alpha N_A \frac{Z(Z+1)}{A} r_e^2 E \ln\left(\frac{183}{Z^{1/3}}\right)$$

which can be rewritten as :

$$-\frac{dE}{dx}^{rad}(e^-) = \frac{E}{X_0}$$

where X_0 is the medium radiation length

then over a path x in the medium, the mean radiated energy of an electron reads :

$$E^{rad}(e^-) = E(1 - e^{-x/X_0})$$

where x is expressed in cm or g/cm^2

and :

$$X_0(g/cm^2) = \frac{716.4\ A(g)}{Z(Z+1)\ln\left(\frac{287}{Z^{1/2}}\right)}$$

In a compound medium :

$$X_o = \left[\sum_i \frac{f_i}{X_0^i}\right]^{-1}$$

where f_i and X_0^i are the mass ratio and the radiation length of element i respectively.

Critical energy : energy at which the ionization stopping power is equal to the mean radiative energy loss of electrons

$$\frac{dE}{dx}^{rad}(E_c)=\frac{dE}{dx}^{ionization}(E_c)$$

$$E_c=\frac{610\ \ MeV}{Z+1.24}$$ for liquids and solids $$E_c=\frac{710\ \ MeV}{Z+0.92}$$ for gas

A better formula that can be used for liquids and solids is : $E_c=2.66\left(\frac{Z}{A(g)}X_0(g/cm^2)\right)^{1.11}\ MeV$

In literature, both the radiation length and the critical energy are tabulated for electrons. For other particles they would scale according to the square of their masses with respect to the electron mass.

medium	***Z***	***A***	***X_0 (g/cm^2)***	***X_0 (cm)***	***E_C (MeV)***
hydrogen	1	1.01	63	700000	350
helium	2	4	94	530000	250
lithium	3	6.94	83	156	180
carbon	6	12.01	43	18.8	90
nitrogen	7	14.01	38	30500	85
oxygen	8	16	34	24000	75
aluminium	13	26.98	24	8.9	40
silicon	14	28.09	22	9.4	39
iron	26	55.85	13.9	1.76	20.7
copper	29	63.55	12.9	1.43	18.8
silver	47	109.9	9.3	0.89	11.9
tungsten	74	183.9	6.8	0.35	8
lead	82	207.2	6.4	0.56	7.4
air	7.3	14.4	37	30000	84
silica (SiO_2)	11.2	21.7	27	12	57
water	7.5	14.2	36	36	83

Electron-positron pair production

At very high energy, direct electron-positron pair production may play an important role.

$$-\frac{dE}{dx}^{pair}=b_{pair}(Z,A,E)\ E$$

Energy loss by photo-nuclear interaction :

example : electro-dissociation of deuteron $e^{-}+d\rightarrow n+p+e^{-}$

$$-\frac{dE}{dx}^{\gamma nucl.}=b_{\gamma nucl.}(Z,A,E)\ E$$

Total stopping power :

$$\frac{dE}{dx}^{tot}=\frac{dE}{dx}^{ionization}+\frac{dE}{dx}^{rad}+\frac{dE}{dx}^{pair}+\frac{dE}{dx}^{\gamma nucl.}$$

which could also be written as :

$$-\frac{dE}{dx}^{tot}=a(Z,A,E)+b(Z,A,E)\ E$$

where a(Z, A, E) is the ionization term and b(Z, A, E) the sum of the Bremsstrahlung, the pair production and the photo-nuclear terms .

Multiple scattering through small angles

A charged particle traversing a medium is deflected many times by small-angles essentially due to Coulomb scattering in the electromagnetic field of nuclei.

This effect is well reproduced by the Molière theory. On both x and y, the angular deflection θ^{proj} of a particle almost follows a Gaussian which is centered around 0 :

$$(\theta^{space})^2 = (\theta_x^{proj})^2 + (\theta_y^{proj})^2$$

$$\theta_{rms}^{proj} = \frac{1}{\sqrt{2}} \theta_{rms}^{space}$$

$$P(\theta^{proj})\, d\theta^{proj} = \frac{1}{\sqrt{2\pi}\,\theta_0} e^{-\frac{1}{2}\left(\frac{\theta^{proj}}{\theta_0}\right)^2} d\theta^{proj}$$

$$P(\theta^{space})\, d\Omega = \frac{1}{2\pi\theta_0^2} e^{-\frac{1}{2}\left(\frac{\theta^{space}}{\theta_0}\right)^2} d\Omega$$

$$\theta_0 = \frac{13.6\, MeV}{\beta p} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right)$$

p particle momentum

X_0 radiation length

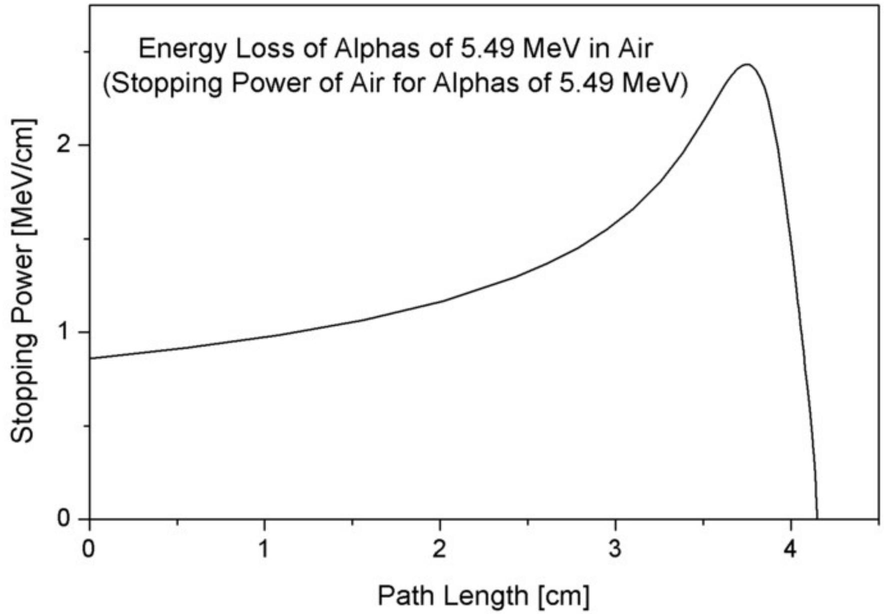
x medium thickness

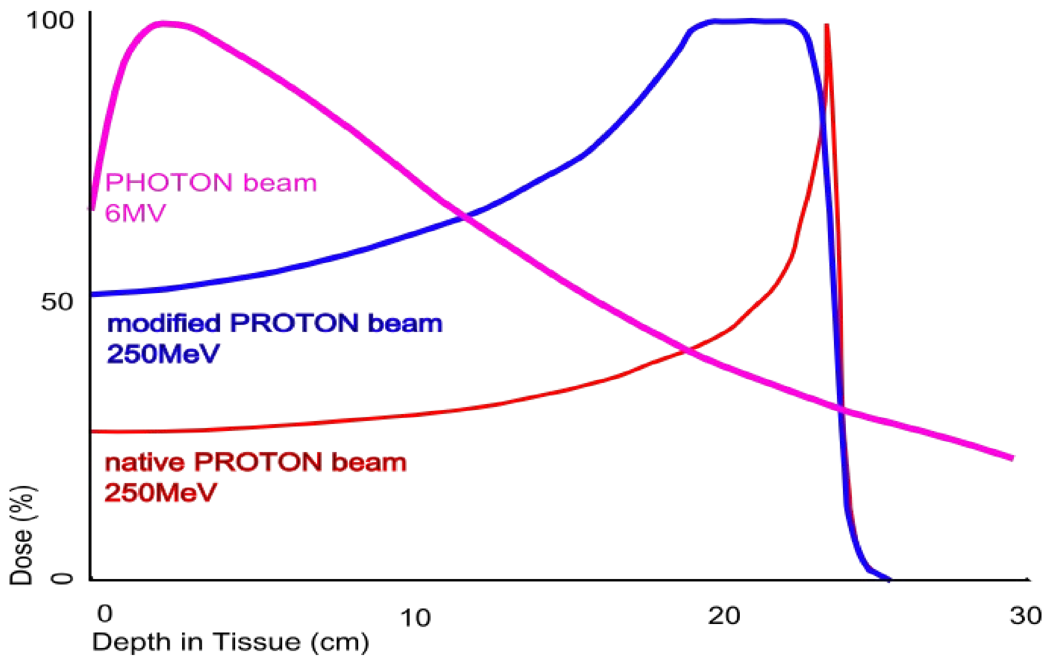
z charge state of incoming particle

Large deflection angles are more probable than what the Gaussian predicts. This results from Rutheford scattering of heavy particles off nuclei. Particles emerging from the medium are also laterally shifted : $X^{rms} = \frac{1}{\sqrt{(3)}} \theta_0 x$

Particle Range in matter

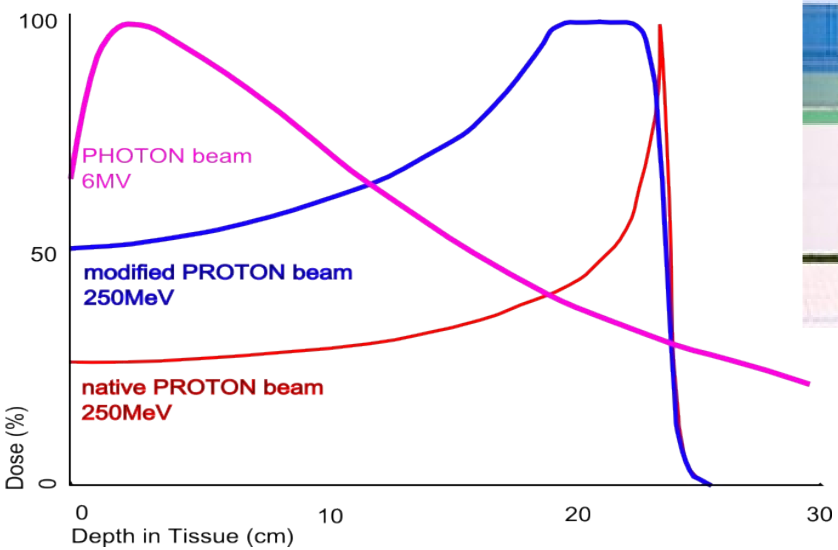
If the medium is thick enough, a particle will progressively decelerate while increasing its stopping power ($\beta^{-5/3}$) until it reaches a maximum (called the Bragg peak).

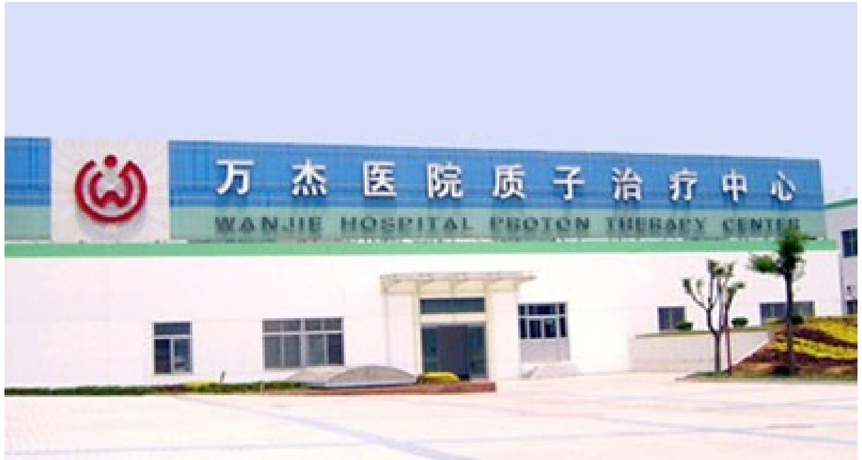




This stopping power profile is used in protontherapy for treating cancerous tumors

Wanjie Proton Therapy Center in Zibo (Shandong Province)





cyclotron delivering 230 MeV protons to treat cancerous tumors

30 centers of that sort around the world.

As the result of the stochastic behavior of particles interacting in matter, it is not possible to enunciate a perfect definition of a reproducible particle range.

Continuous slowing down approximation range :

$$R(T_0) = \int_0^{T_o} \frac{dT}{\frac{dE}{dx}(T)}$$

where T_0 is the incident kinetic energy of the particle.

In practice, the integration is carried out down to 10 eV .

We also use the mean range <R> which corresponds to the distance at which half of the initial particles have been stopped.

If T> 1 MeV , $R \approx \langle R \rangle$.

If only ionization and excitation are used to calculate R(T) (valid for heavy particles with energies < 1 GeV), the following relationship can be used :

$$R_b(M_b, z_b, T_b) = \frac{M_b}{M_a} \frac{z_a^2}{z_b^2} R_a(M_a, z_a, T_b \frac{M_a}{M_b})$$

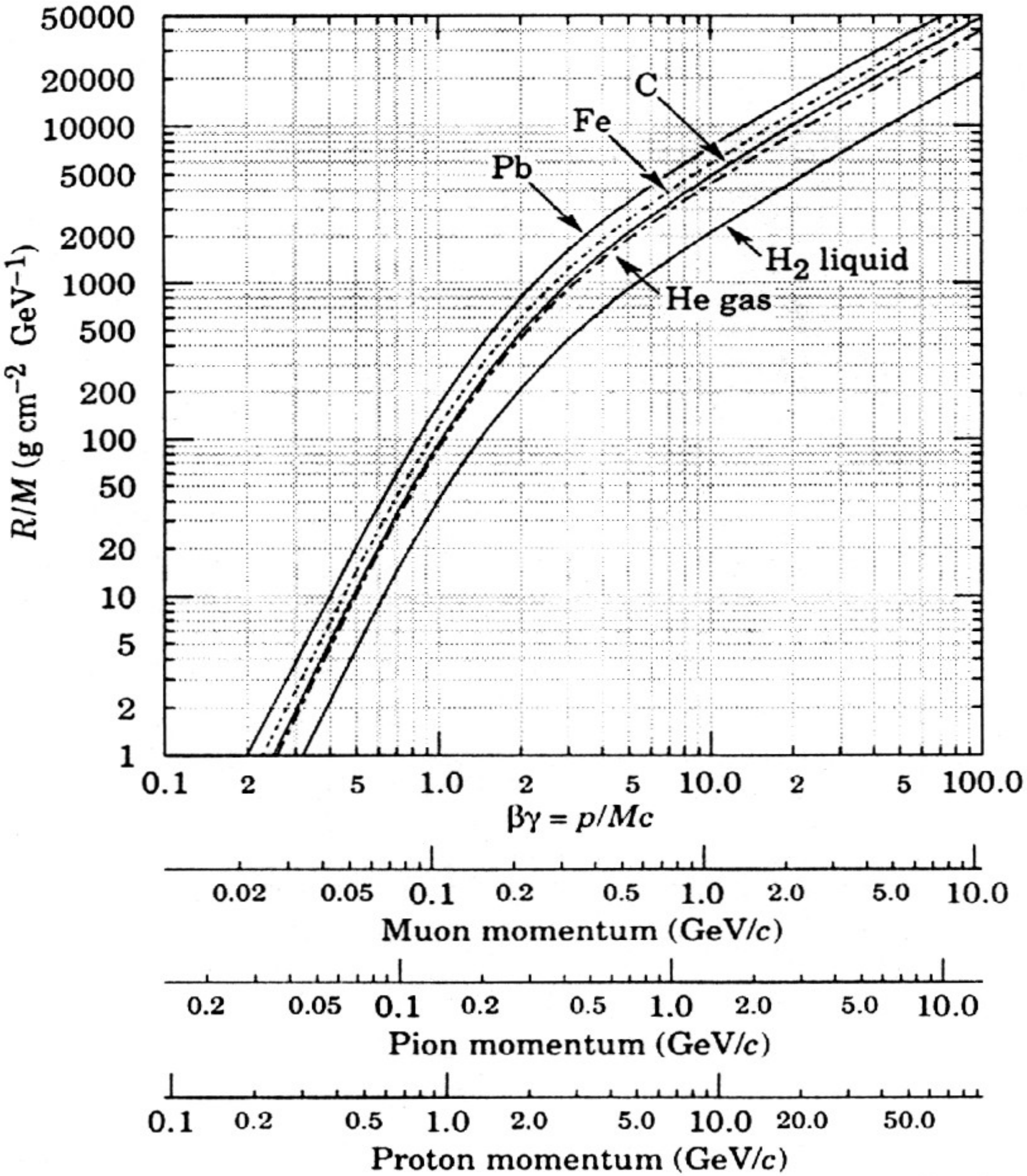
Particle a with z_a , M_a

Particle b with z_b , M_b and kinetic energy T_b

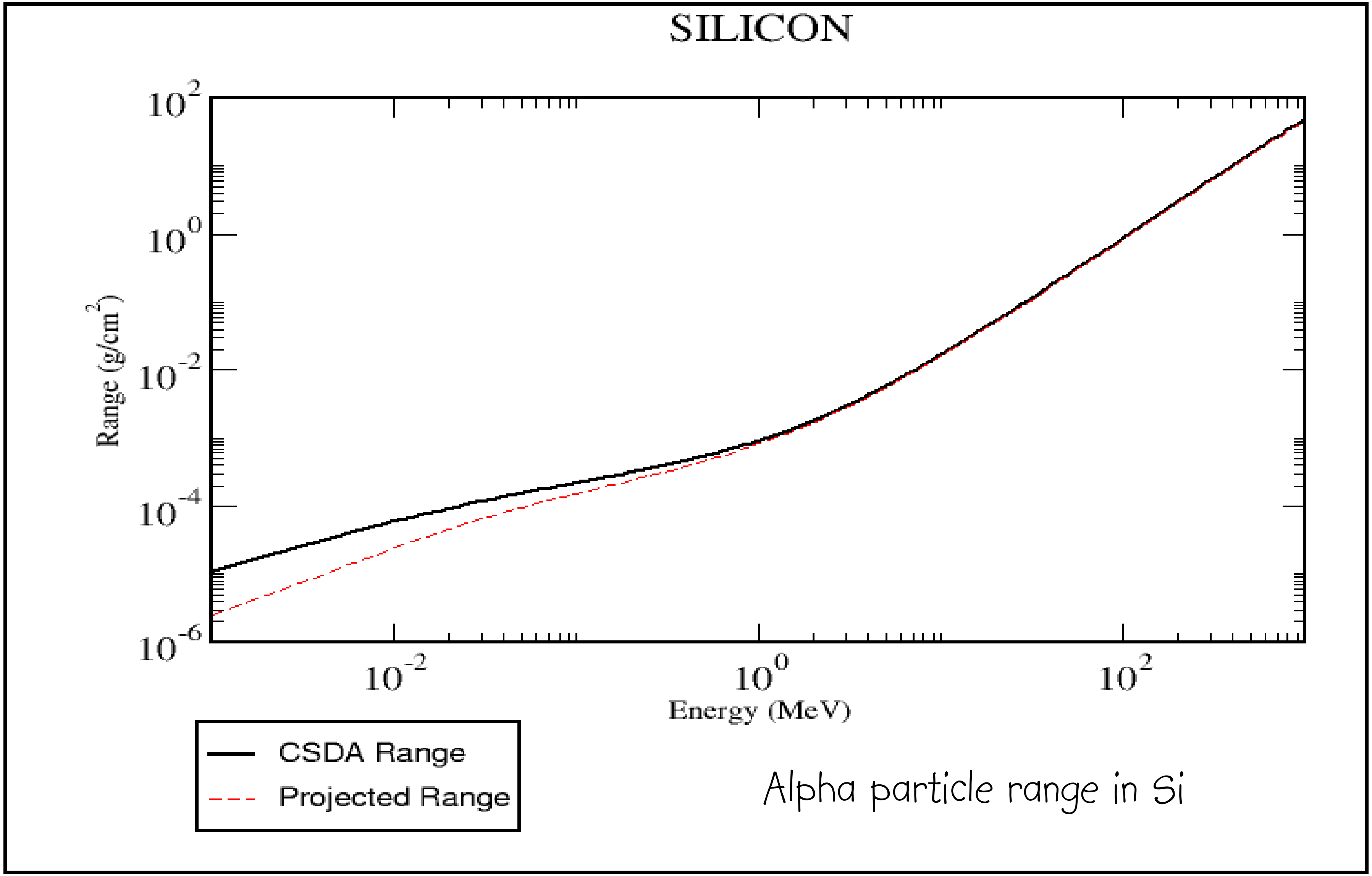
One may also write for a particle of mass M and charge state z carrying a kinetic energy T_0 :

$$R(M, z, T_0) = \frac{M}{z^2} h(T_0/M)$$

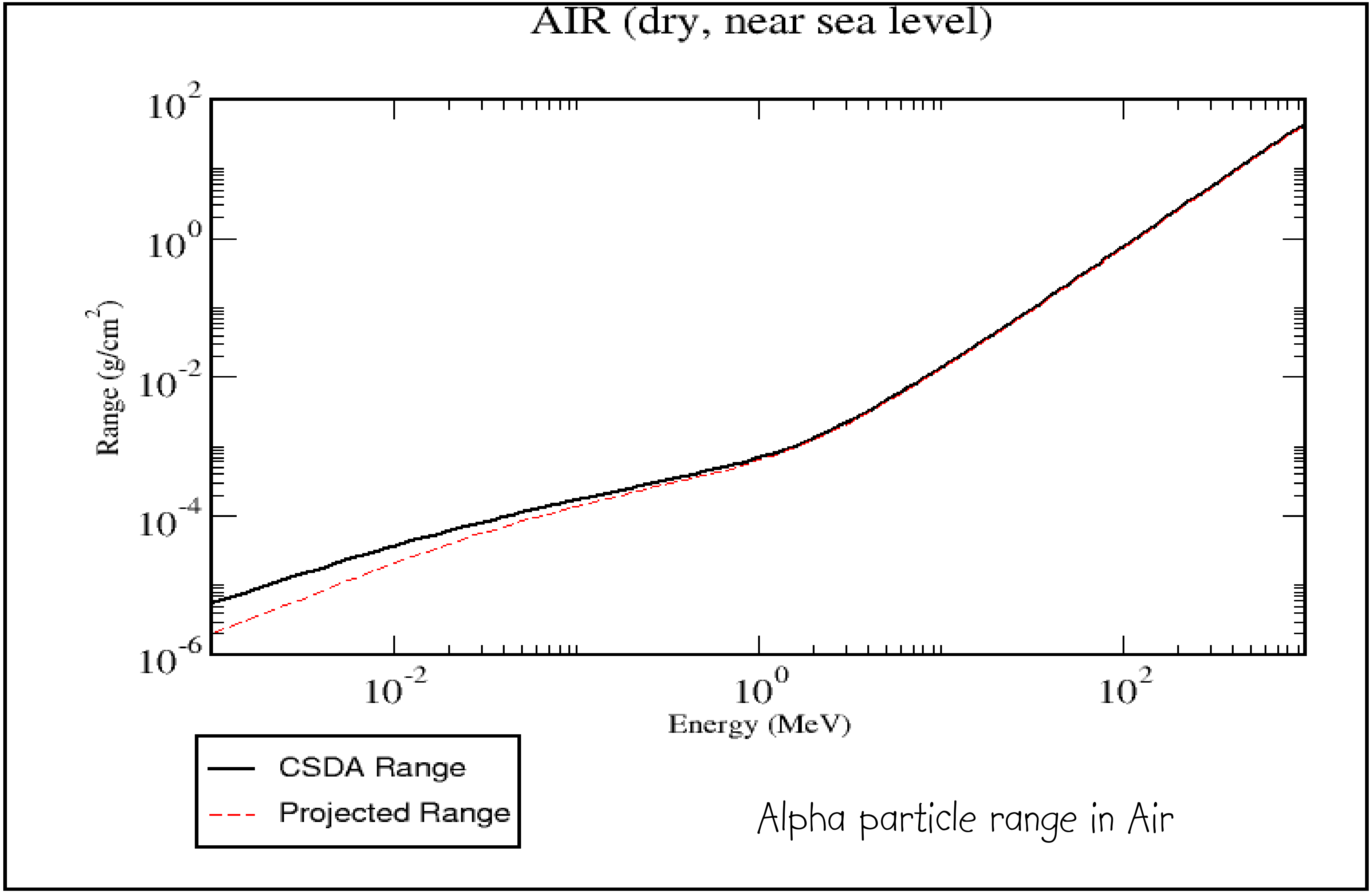
where h is a universal function of the medium (Z, A and I fixed).



universal h function = R



http://physics.nist.gov/PhysRefData/Star/Text

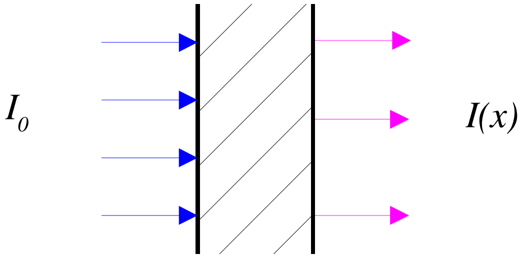

AIR (dry, near sea level)
Range (g/cm²)
10^2
10^0
10^-2
10^-4
10^-6
10^-2
10^0
10^2
Energy (MeV)
CSDA Range
Projected Range
Alpha particle range in Air

Interactions of photons with matter :

Photons are indirectly detected : they first create electrons (and in some cases positrons when interacting by pair production) which subsequently interact with matter.

In their interactions with matter, photons may be absorbed (photoelectric effect or e^+e^- pair creation) or scattered (Compton scattering) through large deflection angles.

As photon trajectories are particularly chaotic, it is impossible to define a mean range. We then proceed with an attenuation law :



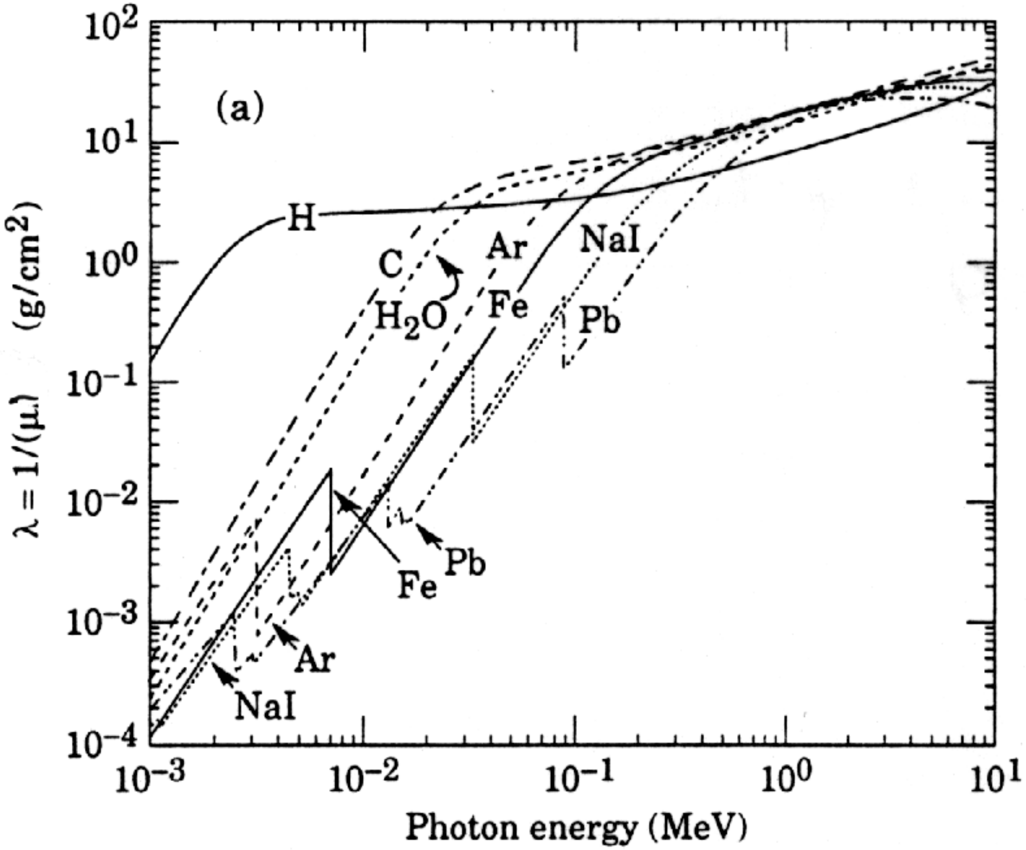
$$I(x) = I_0 e^{-\mu x}$$

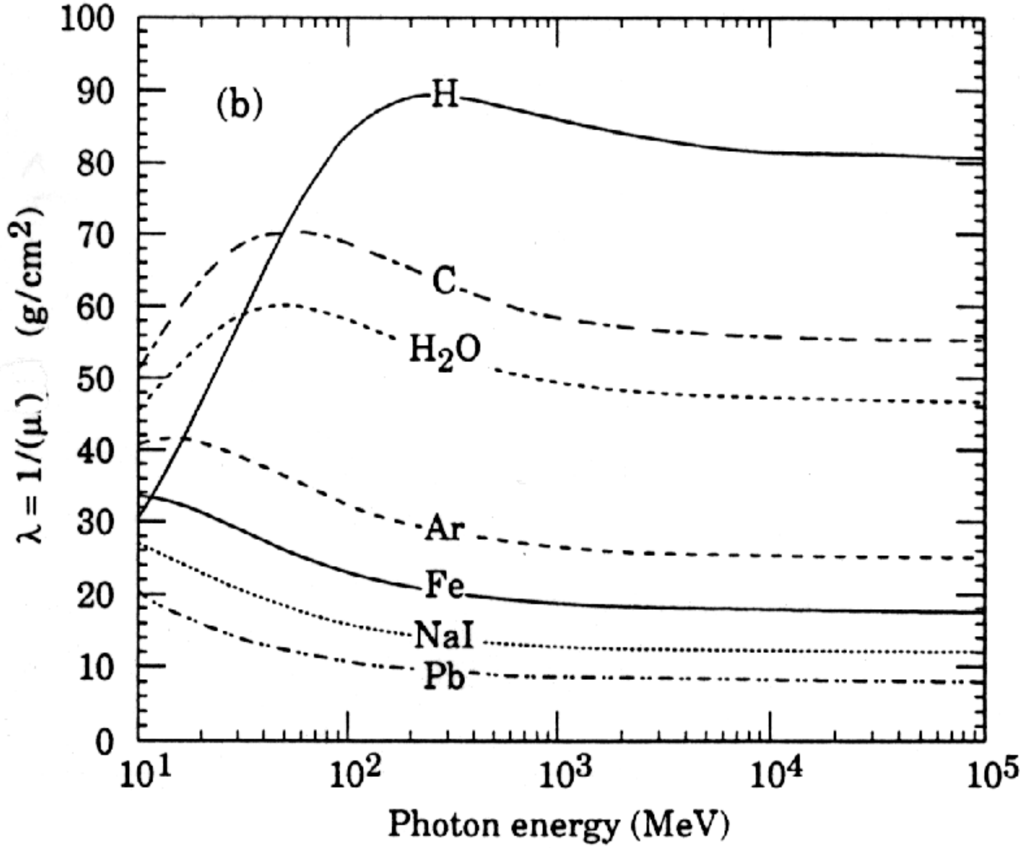
$$\mu = \frac{N_A \sigma_{tot}}{A}$$

where :

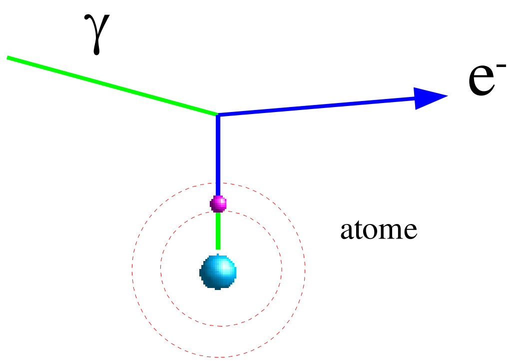
- I_0 is the initial photon beam flux
- $I(x)$ is the photon beam flux exiting the layer of thickness x
- x surface mass density of the layer (g/cm^2)
- μ is the mass attenuation coefficient (cm^2/g)
- σ_{tot} is the total photon cross-section per atom

Photon mean free paths in different media : $\lambda = \frac{1}{\mu}$

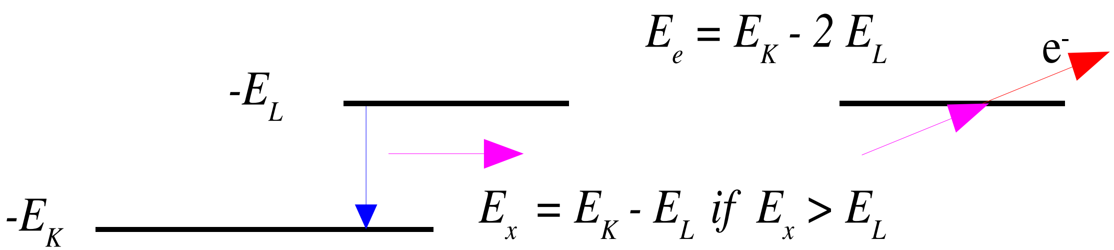




Photoelectric effect :



Because of their proximity to the nucleus, electrons of the deepest shells (K,L,M...) are favored.
Following the emission of a photoelectron, the atom reorganises leading to the production of X rays or Auger electrons.



Production scheme of an Auger electron

Photoelectron energy :

$$E_e = E_\gamma - E_{\text{binding}} \qquad \text{where : } E_{\text{binding}} = E_K \text{ or } E_L \text{ or } E_M \ldots$$

At low energy ($E_\gamma/m_e \ll 1$), but if $E_\gamma \gg E_K$:

$$\sigma^K_{photo} = \left(\frac{32}{\epsilon^7}\right)^{\frac{1}{2}} \alpha^4 \, Z^5 \, \sigma^e_{Th} \quad \text{(per atom)}$$

$\alpha = 1/137$ Fine structure constant

Thomson scattering cross section

$$\epsilon = E_\gamma / m_e$$

$$\sigma^e_{Th} = \frac{8}{3}\pi r_e^2 \text{ with } r_e = \frac{\alpha}{m_e} \text{ classical electron radius}$$

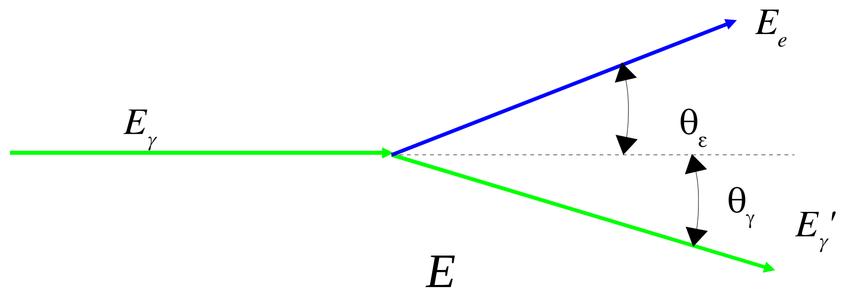
photoelectric cross section strongly increases as Z^5 and decreases as $1/E_\gamma^{3.5}$

At high energy ($E_\gamma/m_e \gg 1$) :

$$\sigma^K_{photo} = 4\pi r_e^2 Z^5 \frac{\alpha^4}{\epsilon}$$

At low energy (E_γ<100 keV) , the photoelectric effect dominates the total photon cross section

Compton scattering : Elastic scattering of a photon off an atomic electron considered as being free (if $E_\gamma > E_{binding}$)



$$\frac{E_\gamma'}{E_\gamma} = \frac{1}{1+\epsilon(1-\cos\theta_\gamma)} \quad with \quad \epsilon = \frac{E_\gamma}{m_e}$$

$$cotg\,\theta_e = (1+\epsilon)\; tg\left(\frac{\theta_\gamma}{2}\right) \qquad \frac{E_\gamma'^{\,min}}{E_\gamma} = \frac{1}{1+2\epsilon}$$

(exercise , show these equations)

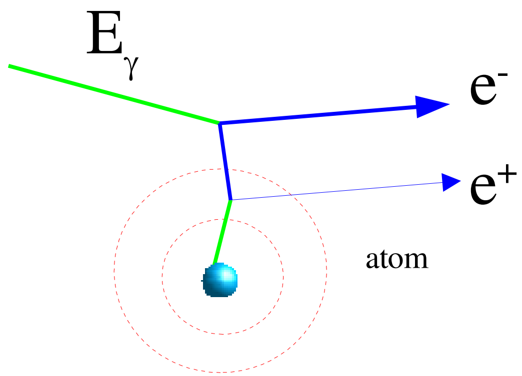
Klein-Nishina cross section :

$$\sigma_c^e = 2\pi r_e^2\left(\left(\frac{1+\epsilon}{\epsilon^2}\right)\left\{\frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon}\ln(1+2\epsilon)\right\} + \frac{1}{2\epsilon}\ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2}\right) \quad (\,per\ electron\,)$$

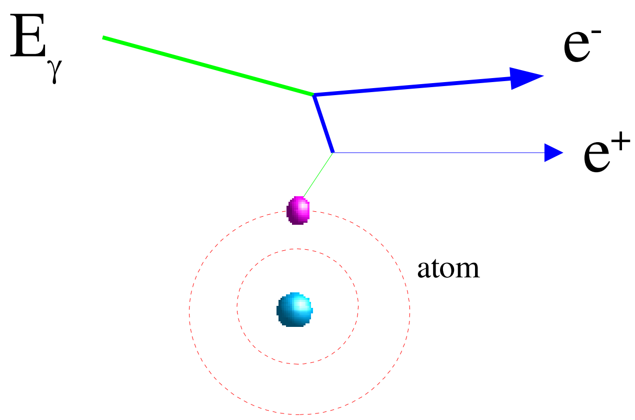
$$\frac{d\sigma_c^e}{d\Omega} = \frac{r_e^2}{2}\frac{1+\cos^2\theta_\gamma}{(1+\epsilon(1-\cos\theta_\gamma))^2}\left(1+\frac{\epsilon^2(1-\cos\theta_\gamma)^2}{(1+\cos^2\theta_\gamma)(1+\epsilon(1-\cos\theta_\gamma))}\right) \quad (\,per\ electron\,)$$

cross-section per atom : $\sigma_c^{atom.} = Z\;\sigma_c^e$

e^+e^- pair production :



$E_\gamma \geqslant 2 m_e$



$E_\gamma \geqslant 4 m_e$

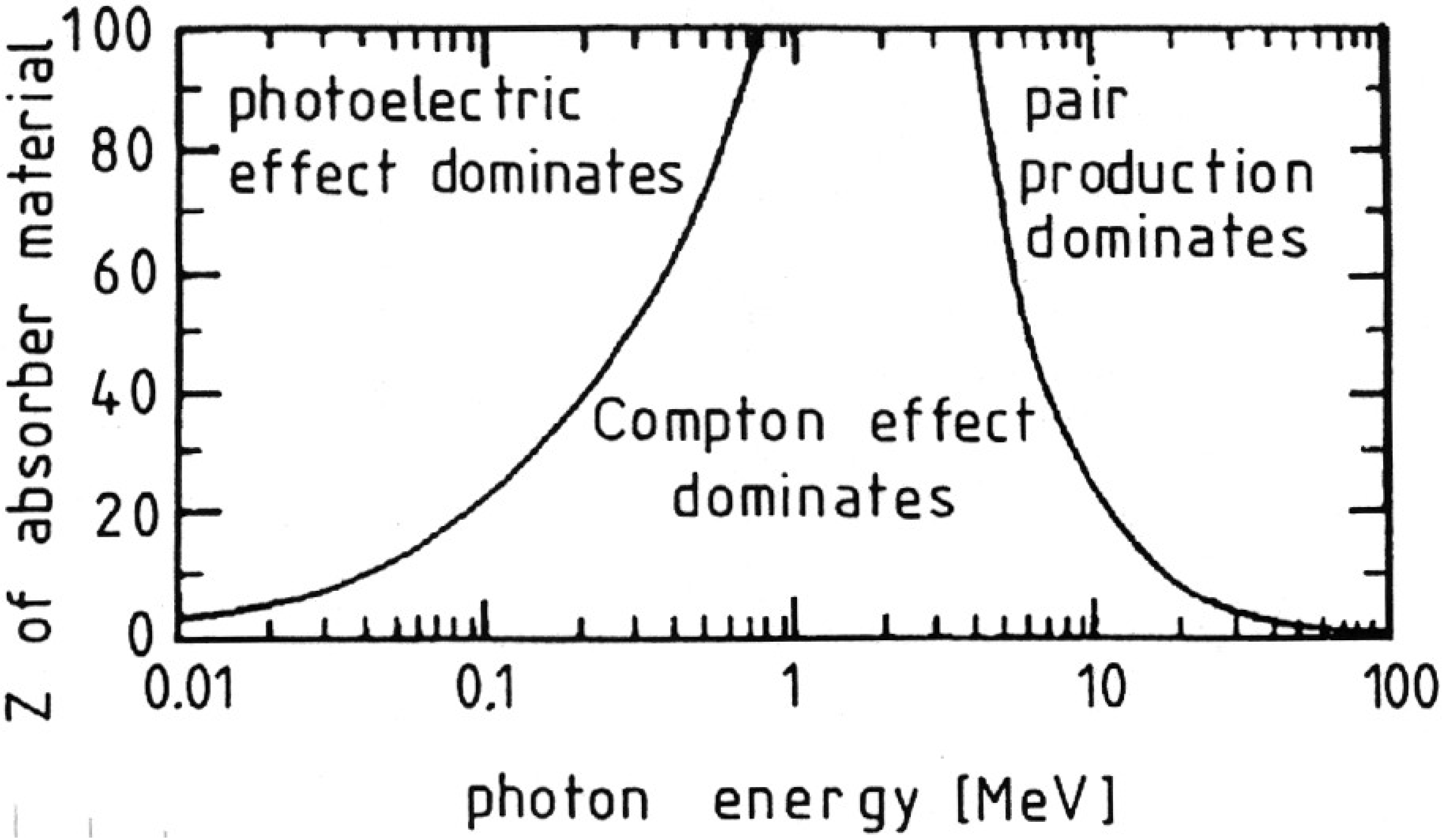
$$1 \ll \epsilon < \frac{1}{\alpha Z^{1/3}} \quad \Rightarrow \quad \sigma_{pair}^{atom.} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln(2\epsilon) - \frac{109}{54} \right)$$

$$\epsilon \gg \frac{1}{\alpha Z^{1/3}} \quad \Rightarrow \quad \sigma_{pair}^{atom.} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{54} \right)$$

In this high energy regime : $\sigma_{pair}^{atom.} \simeq \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$

where X_0 (g/cm^2) is the radiation length

Pair production is the leading effect at high energy


Z of absorber material
100
80
60
40
20
0
0.01
0.1
1
10
100
photon energy [MeV]
photoelectric effect dominates
Compton effect dominates
pair production dominates

Total absorption cross section :

In Compton scattering, photons are not totally absorbed

Let us define a Compton energy scattering cross section :

$$\sigma_{cs}^{atom.} = \frac{E'_{\gamma}}{E_{\gamma}}\,\sigma_{c}^{atom.}$$

And a Compton absorption cross section :

$$\sigma_{ca}^{atom.} = \sigma_{c}^{atom.} - \sigma_{cs}^{atom.}$$

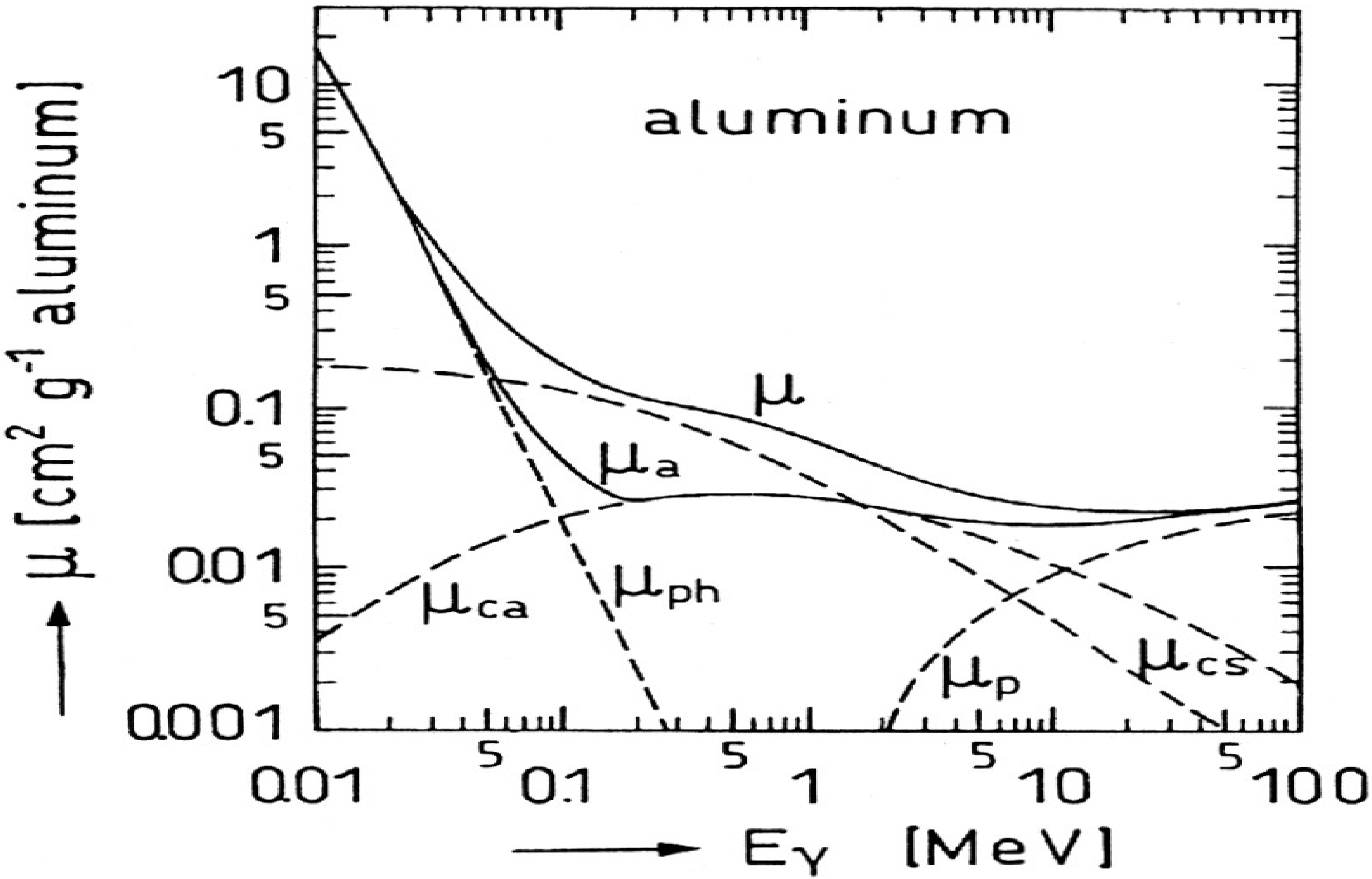
Massic coefficients in cm^2/g :

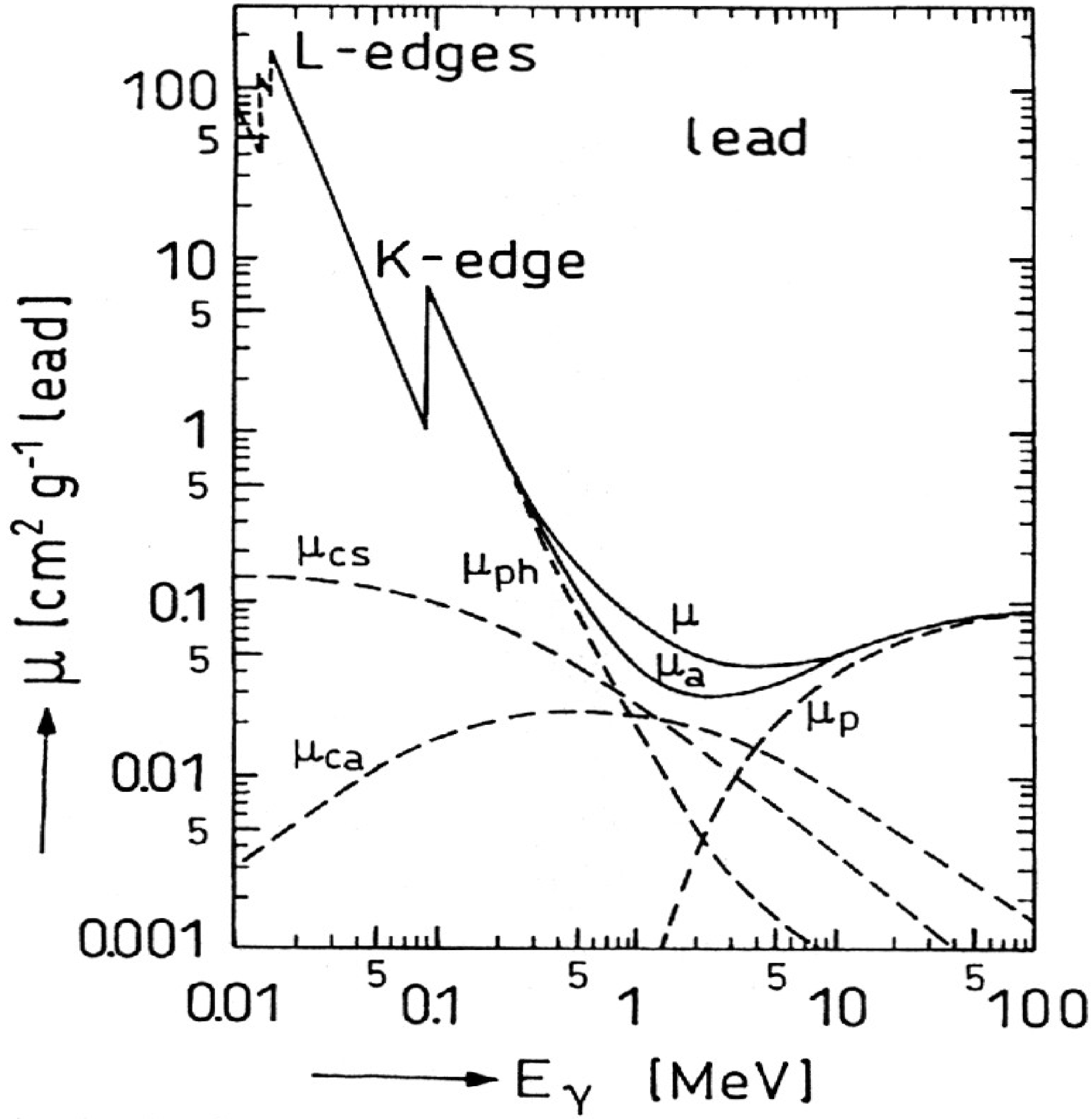
$$\mu_{cs} = \frac{N}{A}\,\sigma_{cs}^{atom.} \quad ; \quad \mu_{ca} = \frac{N}{A}\,\sigma_{ca}^{atom.} \quad ; \quad \mu_{c} = \mu_{cs} + \mu_{ca}$$

$$\mu_{p} = \frac{N}{A}\,\sigma_{pair}^{atom.} \quad ; \quad \mu_{ph} = \frac{N}{A}\,\sigma_{photo}$$

$\mu_{a} = \mu_{ph} + \mu_{p} + \mu_{ca}$ total massic absorption coefficient

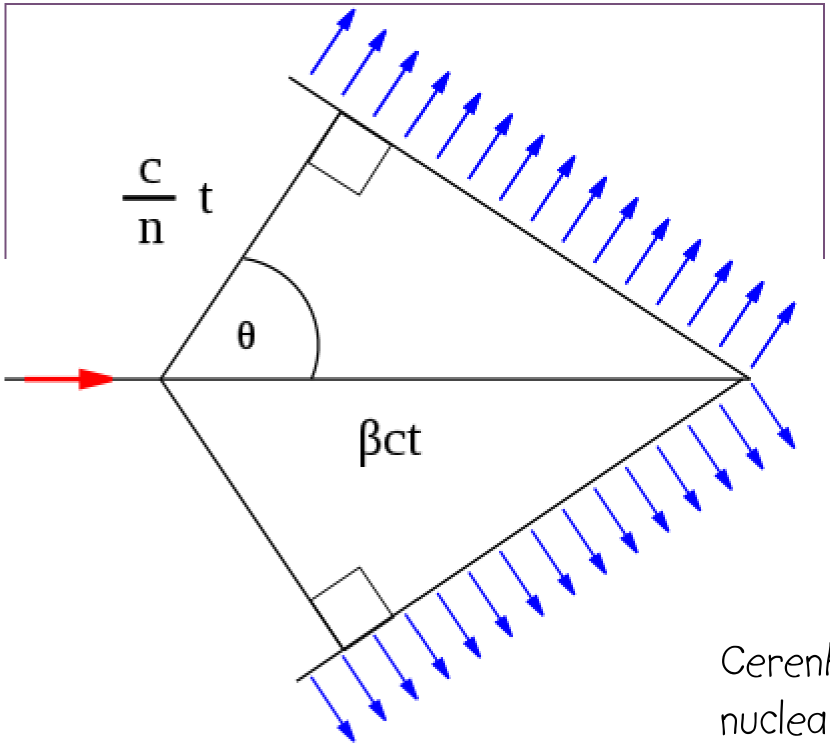
$\mu = \mu_{ph} + \mu_{p} + \mu_{c}$ total massic attenuation coefficient


aluminum
μ
μa
μca
μph
μp
μcs
μ [cm2 g-1 aluminum]
10
5
1
5
0.1
5
0.01
5
0.001
0.01
5
0.1
5
1
5
10
5
100
Eγ [MeV]



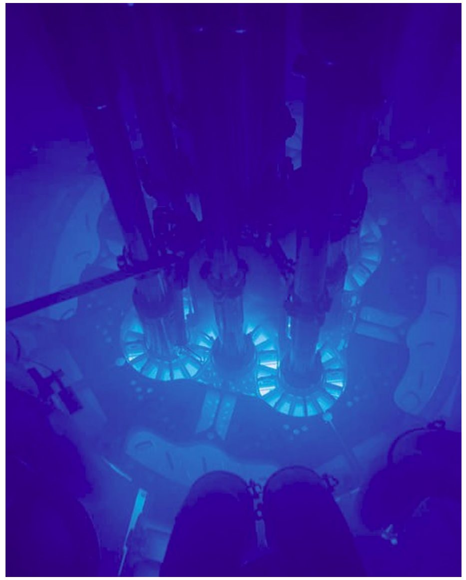
L-edges
lead
K-edge
μ [cm² g⁻¹ lead]
μcs
μph
μ
μa
μp
μca
100
5
10
5
1
5
0.1
5
0.01
5
0.001
0.01
5
0.1
5
1
5
10
5
100
Eγ [MeV]

Cerenkov light emission



When a particle moves faster than the phase velocity of light in the medium, an asymetric polarization of the medium builds up along the longitudinal axis in the vicinity of the particle, that leads to the production of light, which in turn creates a coherent wave front as shown on the picture. It may be understood as the photonic shock wave of a particle that moves faster than c/n where n is the refractive index of the medium.

Cerenkov light emitted by a nuclear reactor (Advanced Test Reactor in ANL)



$$\cos\theta = \frac{c/nt}{\beta ct} = \frac{1}{\beta n}$$

$$\beta > \frac{1}{n}$$

Cerenkov emission has a velocity threshold

The number of photons emitted per unit path length and unit wave length reads :

$$\frac{dN}{dx\,d\lambda}=2\pi\alpha\frac{1}{\lambda^2}\left(1-\frac{1}{\beta^2 n^2}\right)$$

It is strongly peaked at short wave lengths.

The total number of photons per unit path length is then :

$$\frac{dN}{dx}=2\pi\alpha\int_{\beta n>1}\left(1-\frac{1}{\beta^2 n^2}\right)\frac{d\lambda}{\lambda^2}$$

If the variation of n is small over the wavelength region detected then :

$$\frac{dN}{dx}=2\pi\alpha\sin^2\theta\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)$$

e.g in a wavelength interval 350-500 nm (photomultiplier tube), $\frac{dN}{dx}=390\sin^2\theta$ *photons/cm*

dE/dx due to cerenkov light is small compared to ionization loss and much weaker than scintillating output. It can be neglected in energy loss balance of a particle.

This radiation is emitted mostly in the X- ray domain when a particle crosses a boundary between media of different dielectric properties.

The radiation is emitted in a cone at an angle : $\cos\theta = \frac{1}{\gamma}$

The probability of radiation per transition surface is low ~ 1/2 α (fine structure constant)

The energy of radiated photons increases as a function of γ.

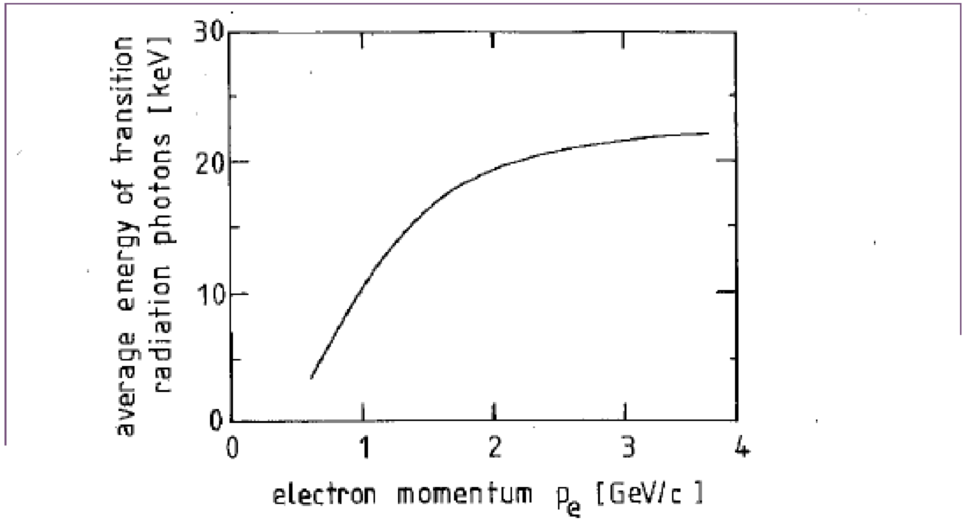
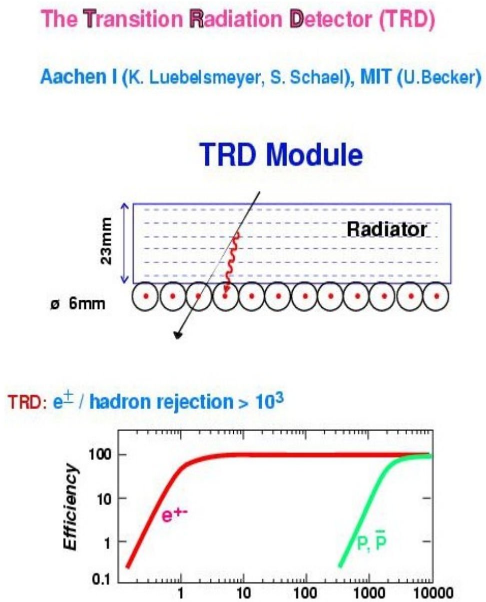


Fig. 6.21. Typical dependence of the average energy of transition radiation photons on the electron momentum for standard radiator arrangements [450].



Deposited energy :

Generally speaking, the energy loss is never equal to the deposited energy as the radiated photons or the secondary particles may escape the medium.

Deposited energy is what generates the signal in a particle detector.

Deposited energy is subjected to large stochastic fluctuations. Remember : Stopping power is the mean energy loss.
If the medium is thin and the number of interactions is small, the deposited energy distribution is asymmetric : it is sometimes called a Landau distribution.
If the medium is thick or the number of interactions is large, the deposited energy distribution tends to a Gaussian.

There are no simple and exact analytical formulae to compute deposited energy.

Nowadays, to estimate the energy deposited in a detector or more generally in a medium we use a Monte-Carlo program which simulates the propagation of the particle through matter : e.g. Geant4 creation of electron-ion pairs :

When the measured signal is a current or a charge liberated through ionizing interactions, it is useful to compute the mean number of created electron-ion pairs :

$$n^{\text{e - ion}} = \frac{\Delta E_{deposited}}{W}$$

where : W is the required mean energy to produce an e-ion pair

W > I (mean excitation and ionization potential)

In most gazes, W ~ 30 eV.

In semiconductor detectors (Ge, Si), W is much lower : e.g. W=3.6 eV for Si and W=2.85 eV for Ge

Hadron collision and interaction lengths :

When dealing with very high energy hadrons, it is somewhat useful to express the total cross-section as :

$$\sigma_T = \sigma_{elastic} + \sigma_{inelastic}$$

Only the inelastic part of the total cross-section is susceptible to induce a hadron shower. It is then useful to introduce two mean lengths :

$$\lambda_T = \frac{A}{N_A \sigma_T} \mathrm{g\ cm}^{-2}$$ called the nuclear collision length

$$\lambda_I = \frac{A}{N_A \sigma_{inelastic}} \mathrm{g\ cm}^{-2}$$ called the interaction length

95% containment of a hadronic shower can be obtained for a thickness of :

$$L_{95\%}(\text{in units of } \lambda_I) \simeq 1 + 1.35 \ln(E(\mathrm{GeV}))$$

Then approximately 10 interaction lengths are needed to contain a 1 TeV hadronic shower.

In high A materials : $\lambda_I > X_0$ which explains why hadron calorimeters are deeper than electromagnetic calorimeters.

6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1. Abridged from `pdg.lbl.gov/AtomicNuclearProperties` by D. E. Groom (2007). Quantities in parentheses are for NTP (20° C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg$1 in brackets are for $(n-1)\times 10^6$ (gases).

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T {g cm^{-2}}	Nucl.inter. length λ_I {g cm^{-2}}	Rad.len. X_0 {g cm^{-2}}	$dE/dx\vert_{\rm min}$ { MeV g^{-1}cm^2}	Density {g cm^{-3}} ({gℓ^{-1}})	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
H_2	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D_2	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
N_2	7	14.0067(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
O_2	8	15.9994(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
F_2	9	18.9984032(5)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	[195.]
Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
Al	13	26.9815386(8)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
Cl_2	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Ti	22	47.867(1)	0.45961	78.8	126.2	16.16	1.477	4.540	1941.	3560.	
Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Ge	32	72.64(1)	0.44053	86.9	143.0	12.25	1.370	5.323	1211.	3106.	
Sn	50	118.710(7)	0.42119	98.2	166.7	8.82	1.263	7.310	505.1	2875.	
Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(4)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T {g cm^{-2}}	Nucl.inter. length λ_I {g cm^{-2}}	Rad.len. X_0 {g cm^{-2}}	$dE/dx\|_{\min}$ { MeV g^{-1}cm^2}	Density {g cm^{-3}} ({gℓ^{-1}})	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
Air (dry, 1 atm)			0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	
Shielding concrete			0.50274	65.1	97.5	26.57	1.711	2.300			
Borosilicate glass (Pyrex)			0.49707	64.6	96.5	28.17	1.696	2.230			
Lead glass			0.42101	95.9	158.0	7.87	1.255	6.220			
Standard rock			0.50000	66.8	101.3	26.54	1.688	2.650			
Methane (CH_4)			0.62334	54.0	73.8	46.47	(2.417)	(0.667)	90.68	111.7	[444.]
Ethane (C_2H_6)			0.59861	55.0	75.9	45.66	(2.304)	(1.263)	90.36	184.5	
Butane (C_4H_{10})			0.59497	55.5	77.1	45.23	(2.278)	(2.489)	134.9	272.6	
Octane (C_8H_{18})			0.57778	55.8	77.8	45.00	2.123	0.703	214.4	398.8	
Paraffin ($CH_3(CH_2)_{n\approx 23}CH_3$)			0.57275	56.0	78.3	44.85	2.088	0.930			
Nylon (type 6, 6/6)			0.54790	57.5	81.6	41.92	1.973	1.18			
Polycarbonate (Lexan)			0.52697	58.3	83.6	41.50	1.886	1.20			
Polyethylene ($[CH_2CH_2]_n$)			0.57034	56.1	78.5	44.77	2.079	0.89			
Polyethylene terephthalate (Mylar)			0.52037	58.9	84.9	39.95	1.848	1.40			
Polymethylmethacrylate (acrylic)			0.53937	58.1	82.8	40.55	1.929	1.19			1.49
Polypropylene			0.55998	56.1	78.5	44.77	2.041	0.90			
Polystyrene ($[C_6H_5CHCH_2]_n$)			0.53768	57.5	81.7	43.79	1.936	1.06			1.59
Polytetrafluoroethylene (Teflon)			0.47992	63.5	94.4	34.84	1.671	2.20			
Polyvinyltoluene			0.54141	57.3	81.3	43.90	1.956	1.03			1.58
Aluminum oxide (sapphire)			0.49038	65.5	98.4	27.94	1.647	3.970	2327.	3273.	1.77
Barium flouride (BaF_2)			0.42207	90.8	149.0	9.91	1.303	4.893	1641.	2533.	1.47
Carbon dioxide gas (CO_2)			0.49989	60.7	88.9	36.20	1.819	(1.842)			[449.]
Solid carbon dioxide (dry ice)			0.49989	60.7	88.9	36.20	1.787	1.563	Sublimes at 194.7 K		
Cesium iodide (CsI)			0.41569	100.6	171.5	8.39	1.243	4.510	894.2	1553.	1.79
Lithium fluoride (LiF)			0.46262	61.0	88.7	39.26	1.614	2.635	1121.	1946.	1.39
Lithium hydride (LiH)			0.50321	50.8	68.1	79.62	1.897	0.820	965.		
Lead tungstate ($PbWO_4$)			0.41315	100.6	168.3	7.39	1.229	8.300	1403.		2.20
Silicon dioxide (SiO_2, fused quartz)			0.49930	65.2	97.8	27.05	1.699	2.200	1986.	3223.	1.46
Sodium chloride (NaCl)			0.55509	71.2	110.1	21.91	1.847	2.170	1075.	1738.	1.54
Sodium iodide (NaI)			0.42697	93.1	154.6	9.49	1.305	3.667	933.2	1577.	1.77
Water (H_2O)			0.55509	58.5	83.3	36.08	1.992	1.000(0.756)	273.1	373.1	1.33
Silica aerogel			0.50093	65.0	97.3	27.25	1.740	0.200	(0.03 H_2O, 0.97 SiO_2)		

To learn more :

- Principles of Radiation Interaction in Matter and Detection, C. Leroy and P.G. Rancoita World Scientific

- Introduction to experimental particle physics, Richard Fernow, Cambridge University Press

- Particle penetration and Radiation effects, P. Sigmund, Springer

- Nuclei and particles, Émilio Segré, W.A. Benjamin

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- Particle detectors, Claus Grupen, Cambridge monographs on particle physics

- Detectors for Particle radiation, Konrad Kleinknecht, Cambridge University Press

- Radiation detection and measurement, G.F. Knoll, J. Wiley & Sons

- Single Particle Detection and Measurement, R. Gilmore, Taylor & Francis

- Radiation detectors, C.F.G. Delaney and E.C. Finch , Oxford Science Publications

- High-Energy Particles, Bruno Rossi, Prentice-Hall

- Introduction to nuclear engineering, John R. Lamarsh, Addison-Wesley Publishing

Interactions of neutrons with matter :

Neutrons are neutral particles which only interact with nuclei.

Neutrons can be absorbed or scattered by nuclei .

With respect to their energy, neutrons can be categorized as follow :

- thermal neutrons : in thermal equilibrium with matter , $\langle E_n \rangle = 3/2\ K\ T$

  $\langle E_n \rangle = 0.038$ eV for $T = 300$ K   K being the Boltzmann constant

  E_0 (most probable energy) = $K\ T = 0.025$ eV for $T = 300$ K
- ultra-cold neutrons $E < 2\ 10^{-7}$ eV
- very cold neutrons $2\ 10^{-7}\ \text{eV} < E < 50\ 10^{-6}$ eV
- cold neutrons $50\ 10^{-6}\ \text{eV} < E < E_0 = 0.025$ eV
- slow neutrons $0.025\ \text{eV} < E < 0.5$ eV
- epithermal neutrons : $0.5\ \text{eV} < E < 1$ keV
- intermediate energy neutrons : $1\ \text{keV} < E < 0.5$ MeV
- fast neutrons : $0.5\ \text{MeV} < E < 50$ MeV
- relativistic neutrons : $50\ \text{MeV} < E$

All neutrons may undergo elastic scattering and radiative capture (emission of photons).

Elastic scattering :

It is mostly used to slow down neutrons , e.g. in a nuclear reactor.

^{1}H , ^{2}H and ^{12}C are the best and preferred moderator nuclei .

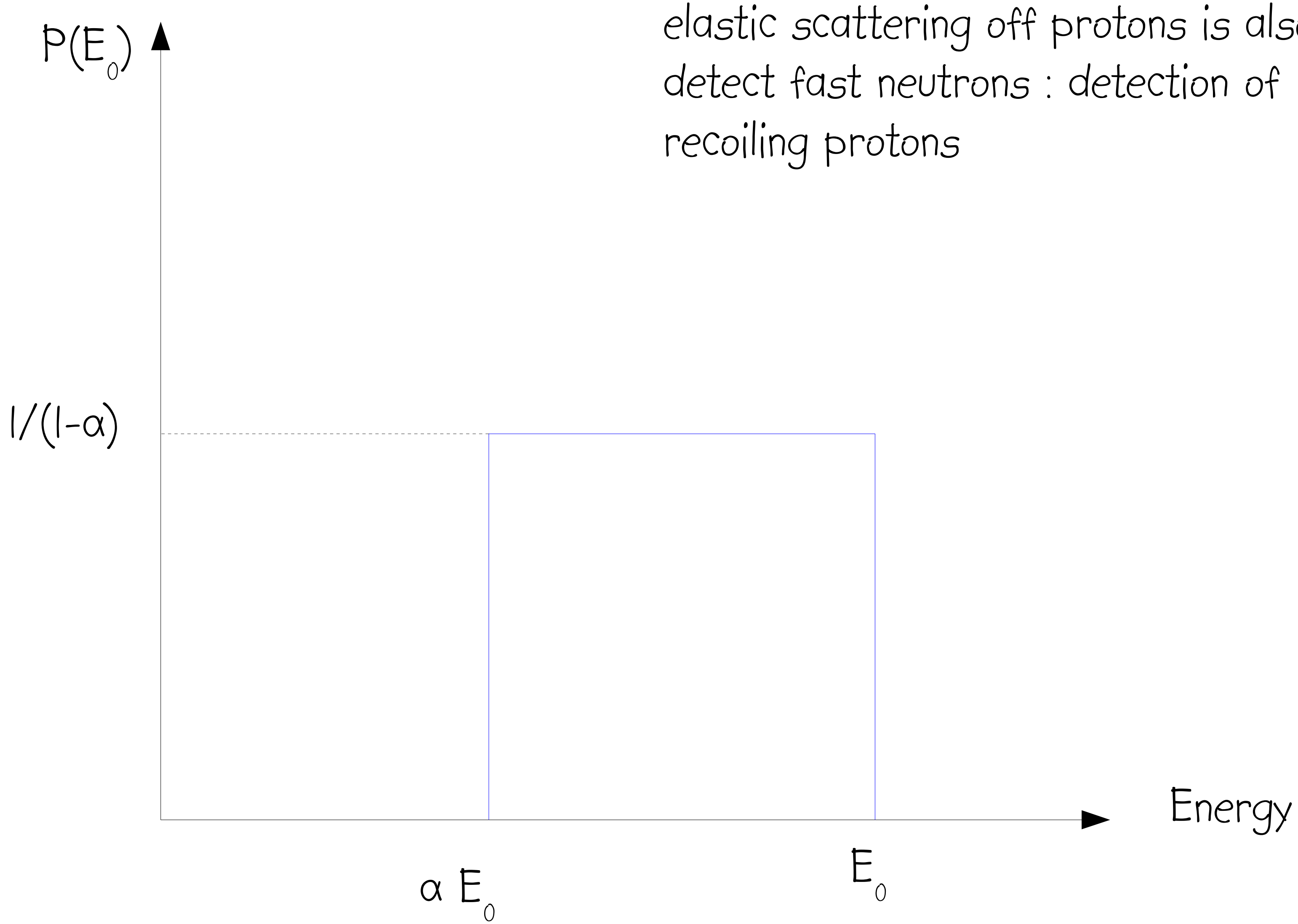
Up to 10 MeV and for some target nuclei (like H), the energy spectrum of elastically scattered neutrons is approximately flat :

The probability of a neutron of mass m_n and incident energy E_0 to be found - after elastic scattering off a nucleus of mass m - in an energy interval dE is :

$$P(E_0)\,dE = \frac{dE}{(1-\alpha)E_0} \quad \text{where} \quad \alpha = \frac{(A-1)^2}{(A+1)^2} \leqslant 1$$

The scattered neutron energy follows : $\alpha E_0 \leqslant E \leqslant E_0$

A is the mass number of the target nucleus elastic scattering off protons is also used to detect fast neutrons : detection of recoiling protons



Energy spectrum of elastically-scattered neutrons
(if the incident neutron energy is less than 10 MeV and the scattering process is approximately isotropic in the center-of-mass-coordinate system)

(n , γ) radiative capture : As neutrons are neutral particles, radiative capture may happen at very low energies (no Coulomb interaction effects)

example : Production of ^{239}Pu in a reactor

$$n + {}^{238}_{92}U \rightarrow {}^{239}_{92}U^{*} \rightarrow {}^{239}_{92}U + \gamma$$

$$ {}^{239}_{92}U \rightarrow {}^{239}_{93}Np + e^{-} + \bar{\nu}_e \rightarrow {}^{239}_{94}Pu + e^{-} + \bar{\nu}_e$$

(n , γ) radiative capture is used to produce artificial radioisotopes in nuclear reactors

It may also be used to detect neutrons and measure neutron fluences (time integrated fluxes).

In general, the capture cross section increases as the inverse of the neutron velocity : the slower the neutrons, the bigger the cross section (Gamow Law). This general behavior may be affected by capture resonances.

(n , γ n')   neutron inelastic scattering :   neutron energy less than a few tens of MeV

gamma decay of nucleus

$$n+{}^{A}_{Z}X \;\rightarrow\; {}^{A+1}_{Z}X^{*} \;\rightarrow\; {}^{A}_{Z}X^{*}+n' \;\rightarrow\; {}^{A}_{Z}X+\gamma+n'$$

decay into neutron + excited nucleus

neutron capture

formation of compound nucleus

(n , p) and (n , α) reactions : A few of these reactions are exoenergetic (produce energy)

The neutron is first captured to produce a compound nucleus which then decays into several products.

$$n + {}^{3}He \rightarrow p + {}^{3}H + 760 \text{ keV} \qquad \sigma = 5400 \text{ barns}$$

$$n + {}^{14}N \rightarrow p + {}^{14}C + 630 \text{ keV} \qquad \sigma = 1.75 \text{ barns}$$

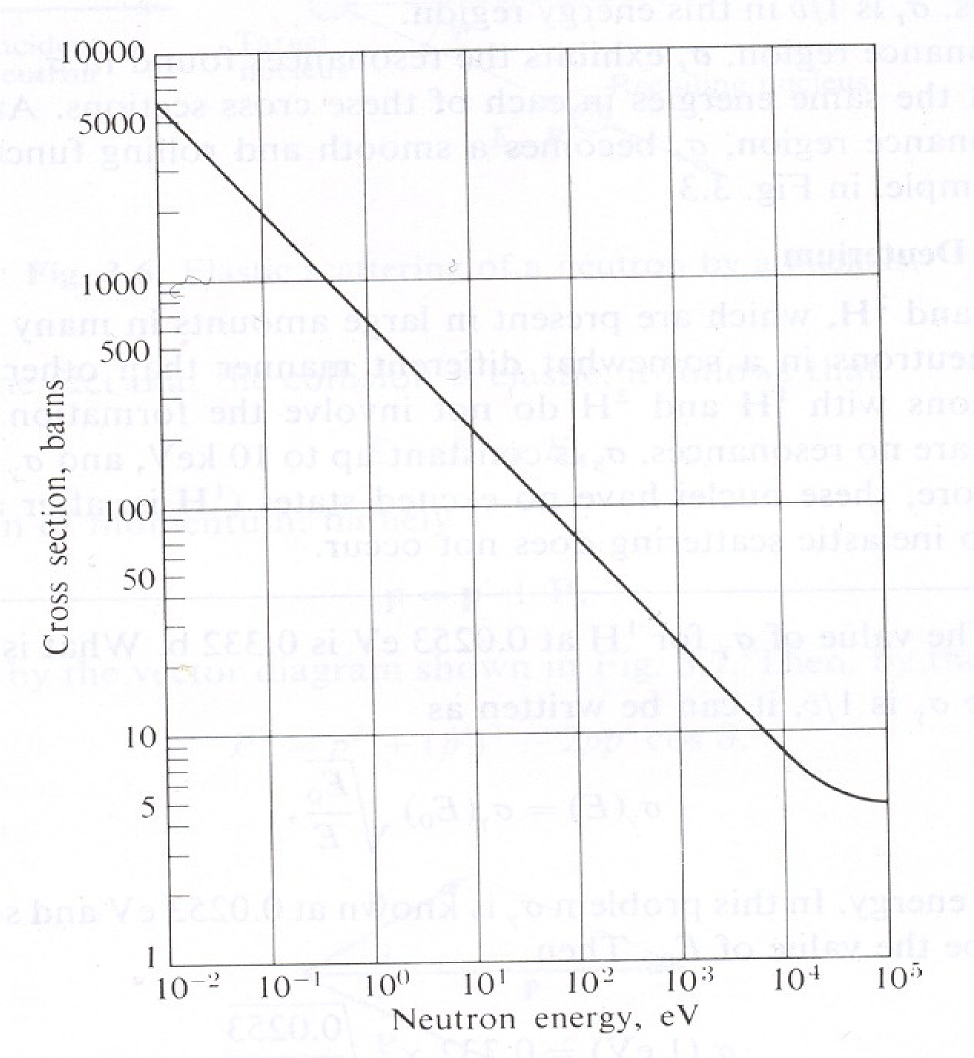
$$n + {}^{35}Cl \rightarrow p + {}^{35}S + 620 \text{ keV} \qquad \sigma = 0.8 \text{ barn}$$

$$n + {}^{6}Li \rightarrow \alpha + {}^{3}H + 4782 \text{ keV} \qquad \sigma = 940 \text{ barns}$$

$$n + {}^{10}B \rightarrow \alpha + {}^{7}Li + 2791 \text{ keV} \qquad \sigma = 4000 \text{ barns}$$

These reactions (in particular on ^{3}He , ^{6}Li, ^{10}B) are used to detect low energy neutrons.

In general, the capture cross section increases as the inverse of the neutron velocity : the slower the neutrons, the bigger the cross section (Gamow Law). This general behavior may be affected by capture resonances.



Cross section of $^{10}B(n,alpha)^{7}Li$

neutron-induced fission :

fission fragments

$$n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X^{*} \rightarrow N + M + \text{a few neutrons} + 200\,MeV$$

For some odd neutron number nuclei (odd N), fission may occur at all neutron energies and in particular at very low energy . This is the case of :

^{233}U , ^{235}U , ^{239}Pu , ^{241}Pu

For other nuclei, fission only takes place above a neutron energy threshold, e.g.

^{238}U : 0.9 MeV , ^{232}Th : 1.3 MeV

Apart from its well-known application for massive energy production, fission may be used to detect neutrons.

Neutron cross sections :

Macroscopic cross section is defined as : $\Sigma = n\,\sigma$

σ: microscopic cross section

n: number of nuclei per unit volume

Σ [cm^{-1}]

$$\Sigma_{tot} = \Sigma_{\text{elastic scattering}} + \Sigma_{\text{absorption}} + \Sigma_{\text{inelastic scattering}}$$

As in the case of gammas, we can use an attenuation law :

$$I(x) = I_0 e^{-\Sigma_{tot} x}$$

I_0: initial neutron flux

x: distance traversed by neutrons

$\lambda = \frac{1}{\Sigma_{tot}}$ being the neutron mean free path in the considered medium