Elementary constituents of matter and their interactions in the Large Hadron Collider (LHC) era

Lectures of the physics doctoral school of Grenoble

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Foreword

These courses are intended for students with a master degree in physics, who never followed any lectures in particle physics. They provide a brief introduction to the phenomenological physics program of the LHC.

The objectives of the construction of the standard theory (the Standard Model) are briefly outlined in a simplified manner in a first part which is accessible to all. Interactions and elementary particles are introduced starting from the point of view of their desired symmetries (U (1), SU(2) and SU(3)).

Some needed notions of special relativity, kinematics, collision cross sections and quantumfield theory are presented in a very simplified manner.

From the symmetries U (1), SU (2) and SU (3), we then construct the standard model of elementary particle interactions in its most compact covariant form.

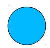
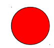
These courses include a brief analysis of the phenomenological content of the electroweak standard model. It continues with a brief presentation of the treatment of masses of elementary particles and concludes with some experimental results concerning the weak bosons, the top quark and the Higgs boson.

It does not take much to make a world !

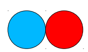
Music : 7 notes and 2 accidentals (flat and sharp)
- positioned on staves according to their frequency
- bound in chords

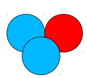
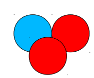
Physical Universe : 12 elementary particles
and 4 interactions
- ordered in multiplets
- bound states (nuclei, atoms, molecules,
crystals, cosmic bodies ...)

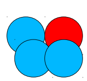
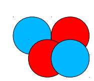
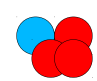
Isotopic Spin : Isospin

A=1  neutron Q= 0  proton Q = |e| $m_{proton} / m_{neutron} = 0.9986 \approx 1$

Same object (nucleon) that "spins" around its own axis in an "abstract" space (isotopic spin space) with two possible isotopic spin projections : ½ and -½
Proton I = ½ , Neutron I = - ½

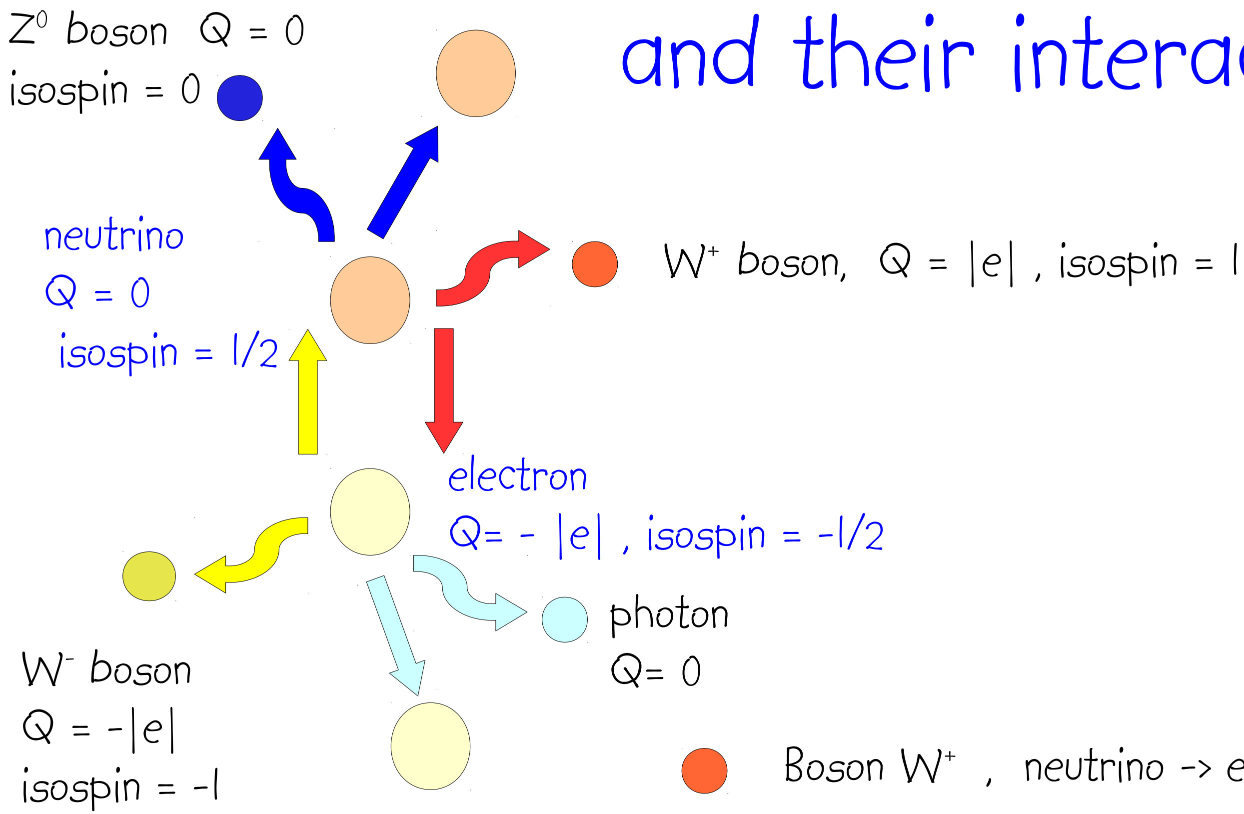
A=2  ${}^{2}_{1}H$ I=0

A=3  ${}^{3}_{1}H$ I= - ½  ${}^{3}_{2}H_{e}$ I= ½ $m_{{}^{3}_{1}H} / m_{{}^{3}_{2}H_e} \sim 1$

A=4  ${}^{4}_{1}H$ I= -1  ${}^{4}_{2}H_{e}$ I= 0  ${}^{4}_{3}L_{i}$ I= 1

$$m_{{}^{4}_{1}H} : m_{{}^{4}_{2}H_e} : m_{{}^{4}_{2}H_e} = 1.006 : 1 : 1.005$$

Elementary particles and their interactions



Boson W^+ , neutrino -> electron

Boson Z^0 , neutrino -> neutrino , electron -> electron

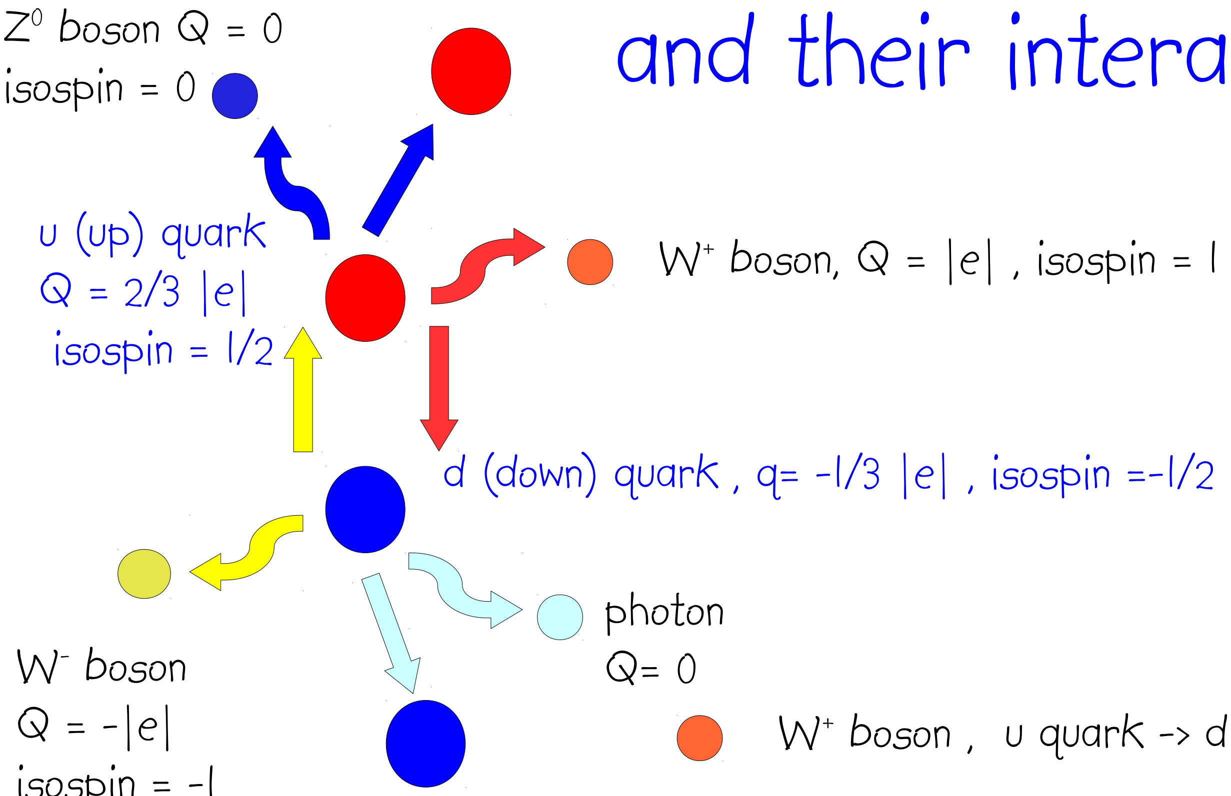
Boson W^- , electron -> neutrino

Weak interaction

photon , electron -> electron

Electromagnetic interaction

Elementary particles
and their interactions



W^+ boson , u quark -> d quark

Z^0 boson , u quark-> u quark ; d quark -> d quark

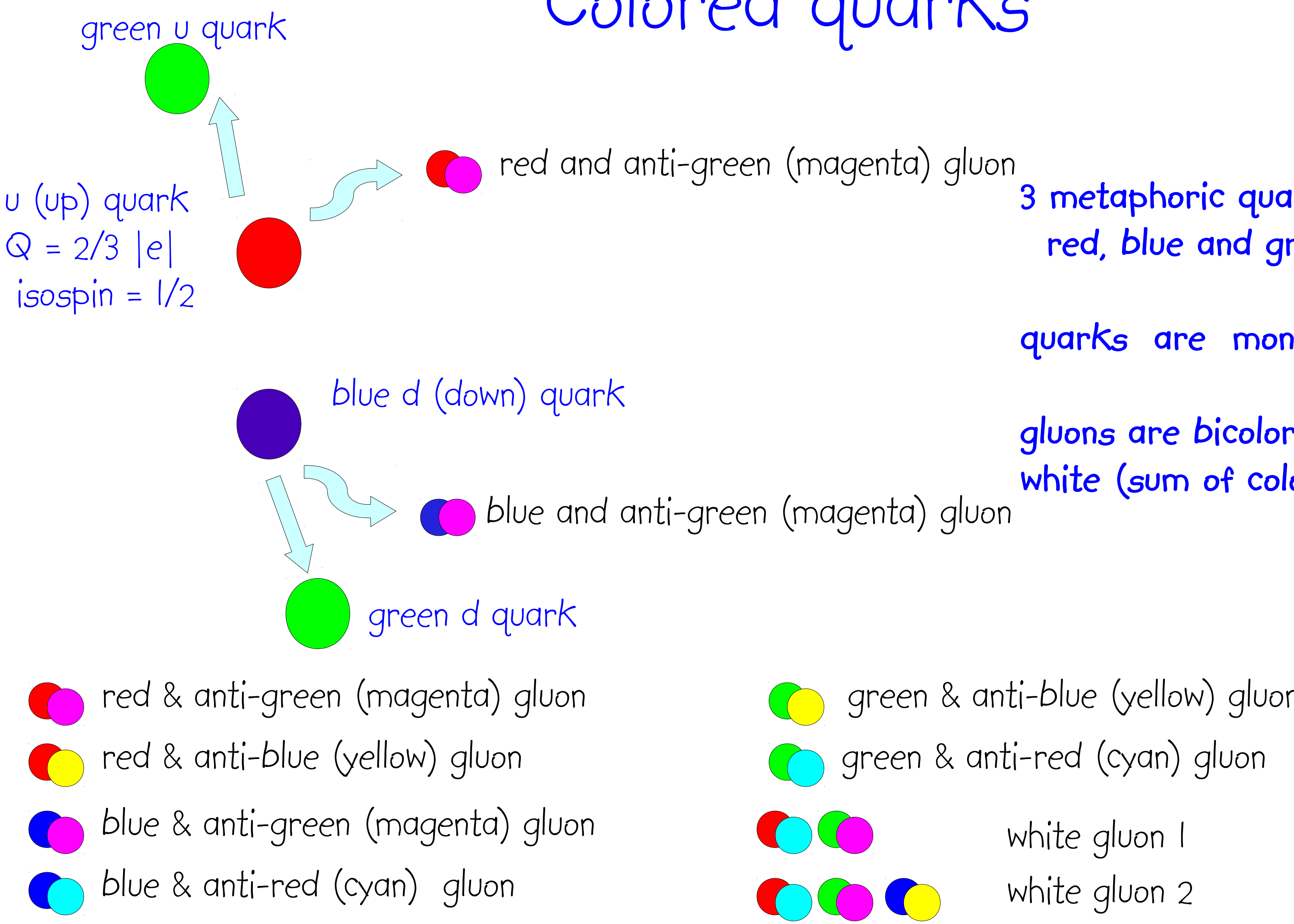
W^- boson , d quark-> u quark

Weak interaction

photon , u quark -> u quark ; d quark -> d quark

Electromagnetic interaction

Colored quarks



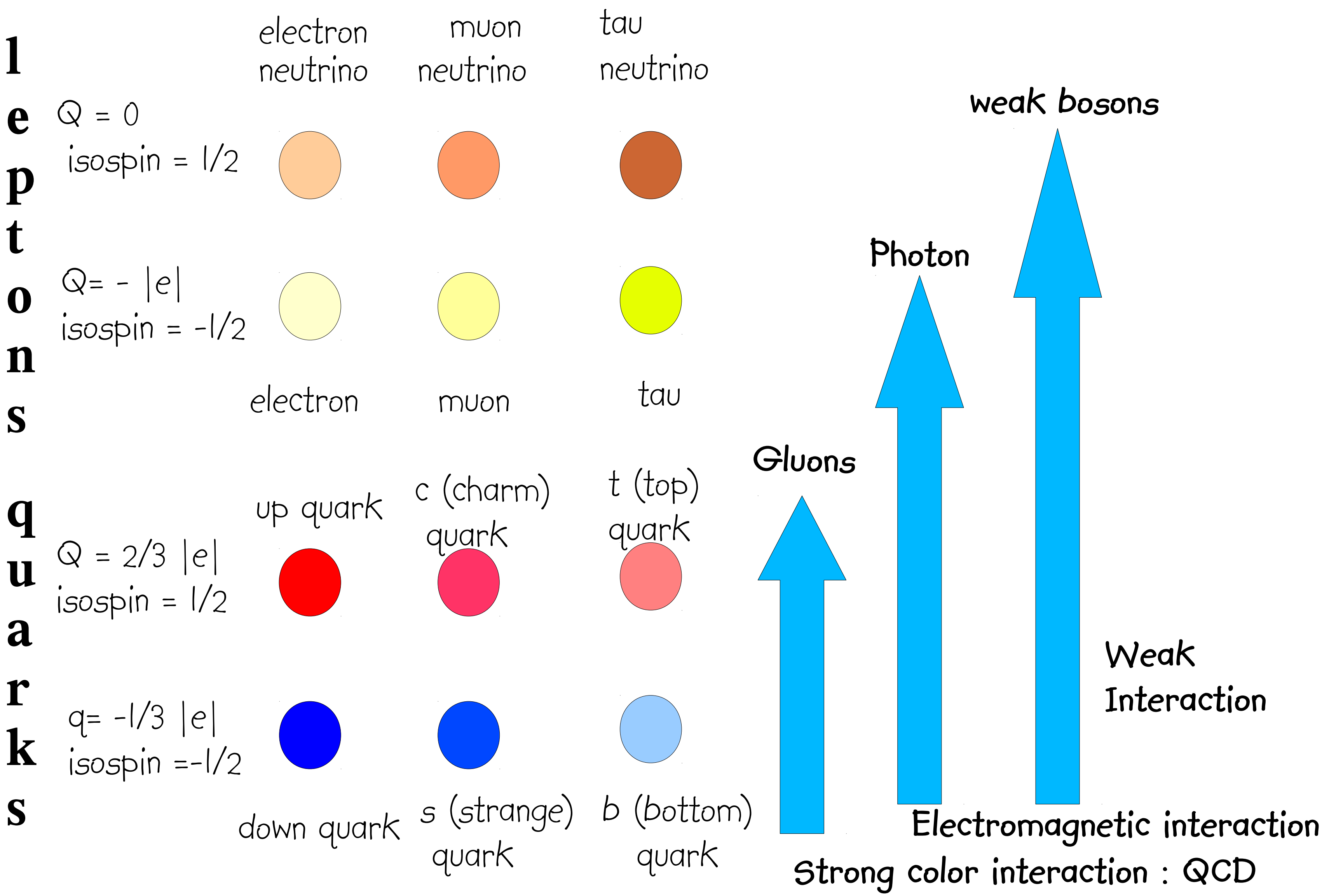
3 metaphoric quantum colors : red, blue and green

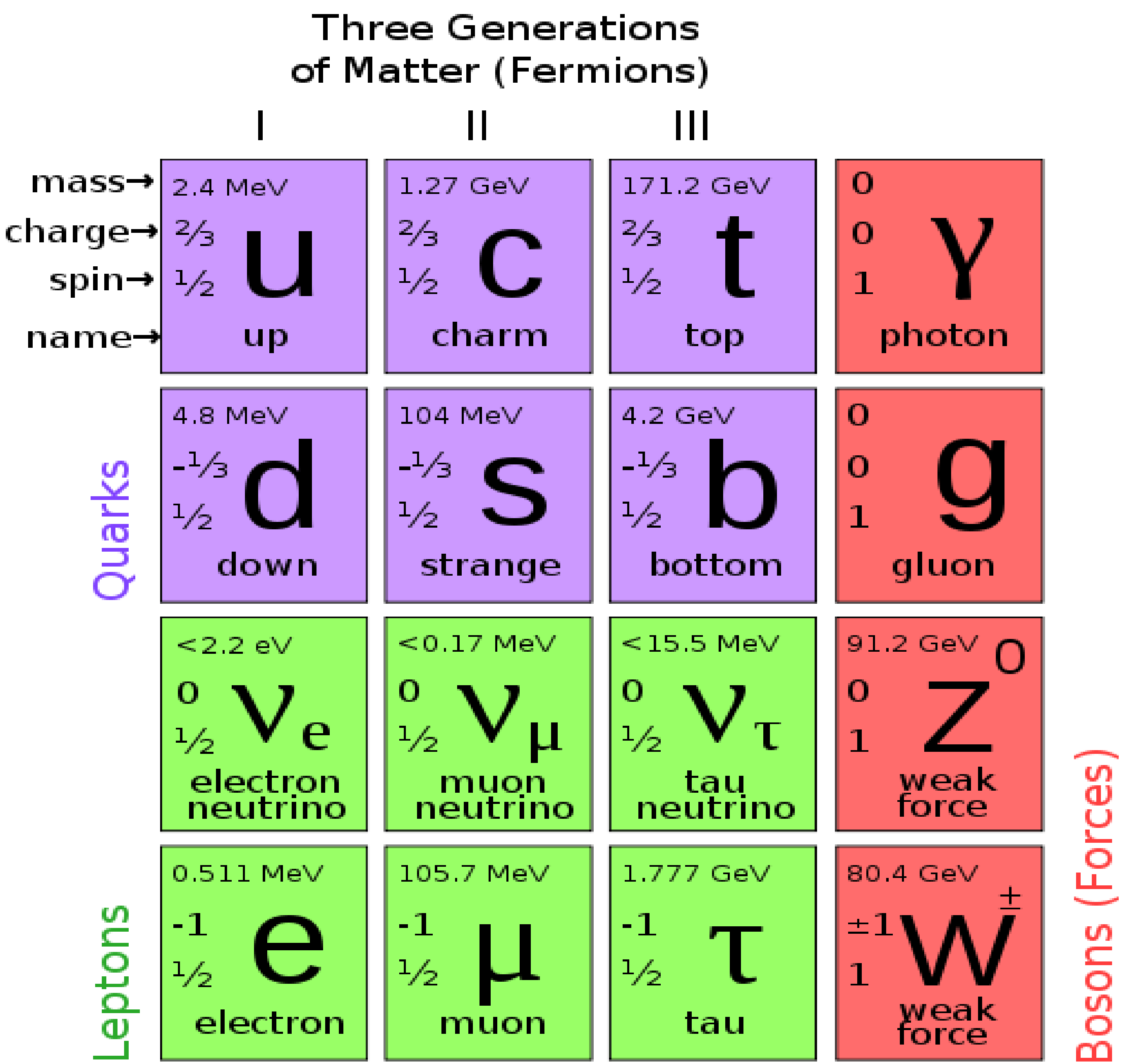
quarks are monochromatic

gluons are bicolored or white (sum of colors)

Strong color Interaction : Quantum Chromo Dynamics (QCD)

Elementary constituents





To learn more on elementary particle properties : http://pdg.lbl.gov/

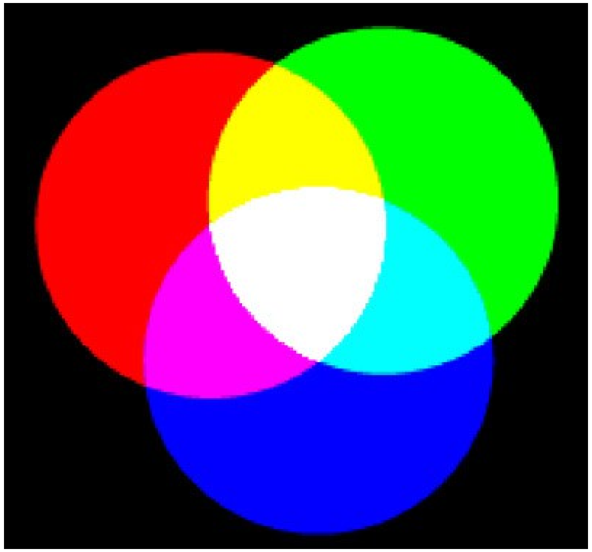
Hadrons

These are all bound states consisting of quarks and anti-quarks. Hadrons are not colored : they are color singlets : in other words, they are white.

One can show that only the systems containing a quark and an antiquark ($q\bar{q}$) or three quarks (qqq) respect this principle.

This situation is analogous to what is obtained in the additive color synthesis of light, where white is obtained by adding the three primary colors (RBG), or by mixing one of the three primary colors with its complementary color (G and M, R and C, B and Y).

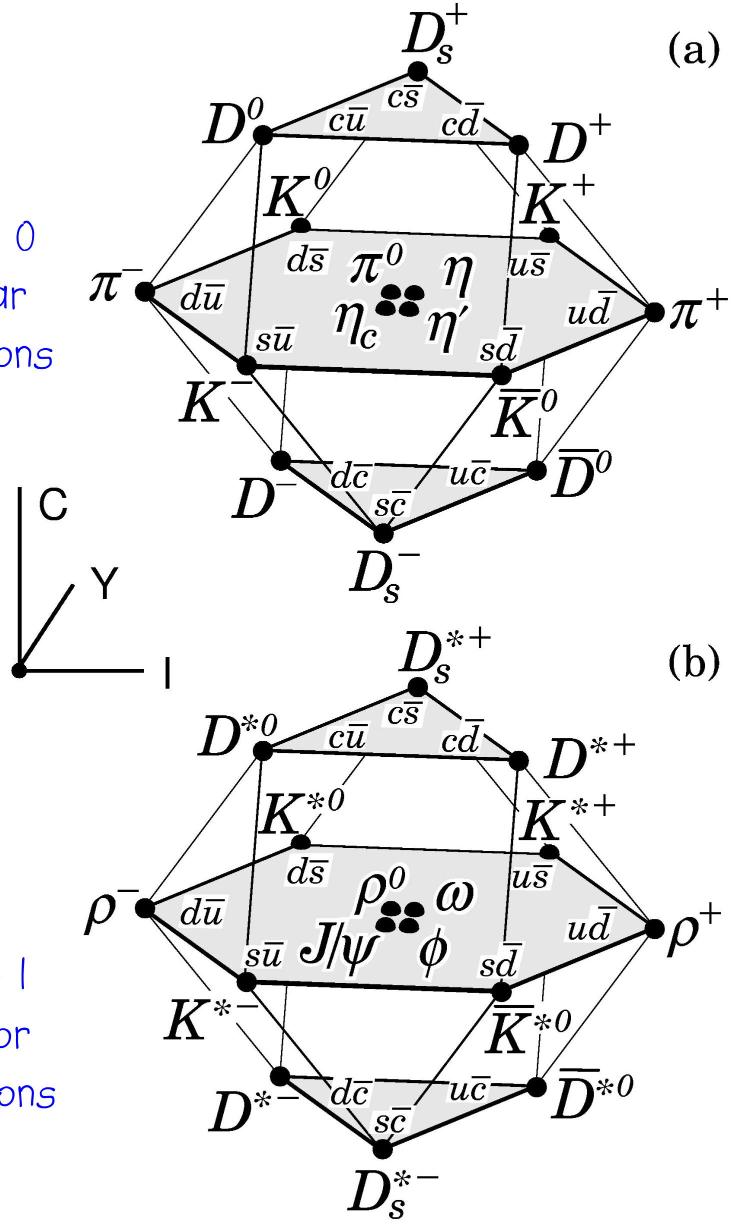
$q\bar{q}$ systems are called mesons, while
qqq systems are called baryons.



Hadrons made of u, d, s and c quarks

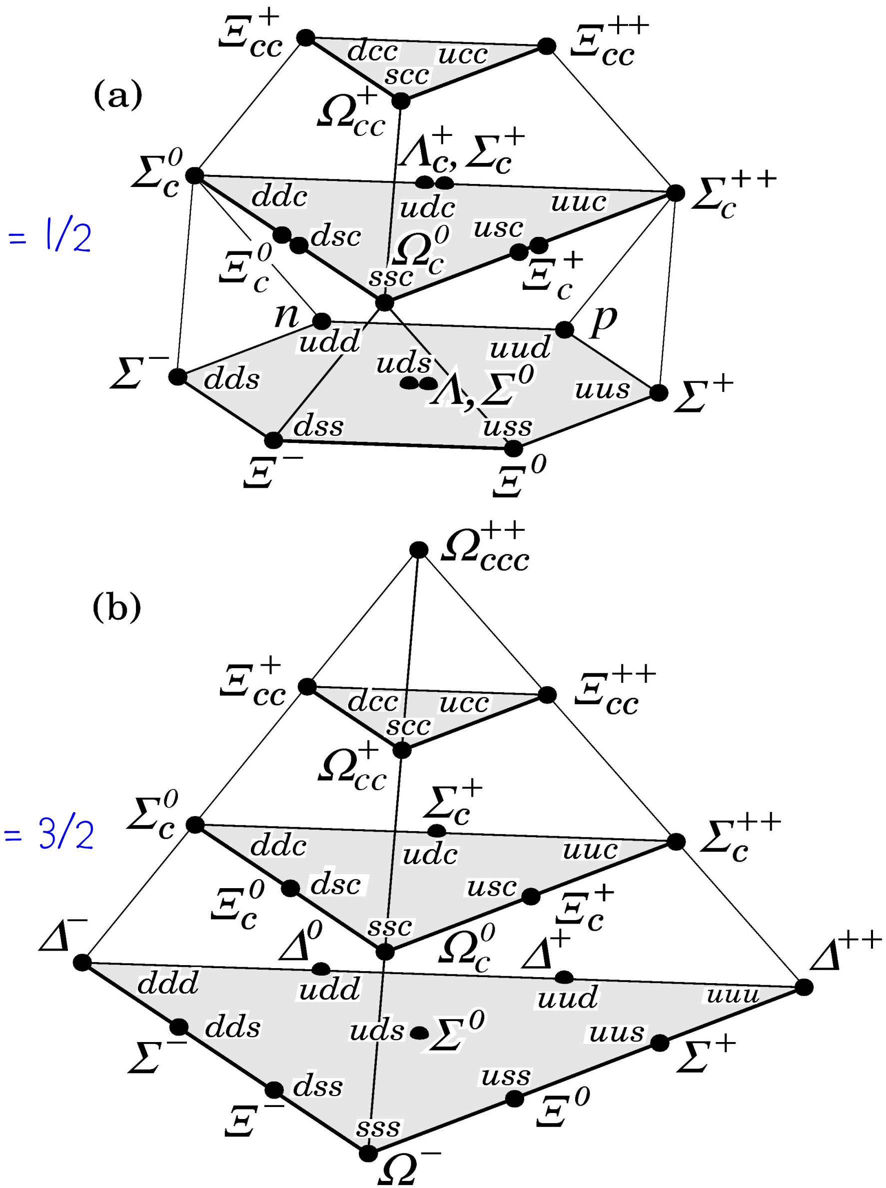
Spin 0
scalar
mesons

Spin 1
vector
mesons



spin = 1/2

spin = 3/2



Mesons

Baryons

If all these quarks had the same mass, and if one neglects their electromagnetic interaction, all properties (mass, spin ...) of these hadrons in a given multiplet would be identical.

Natural Units

[E] en GeV = $1.6\ 10^{-10}$ J

[d] en fm = 10^{-15} m

$\hbar = 6.582\ 10^{-22}$ MeV s

$c = 299792458$ m s^{-1}

$\hbar c = 197.3$ MeV fm

$k = 8.617\ 10^{-5}$ eV K^{-1}

$(\hbar c)^2 = 0.389$ GeV^{-2} mb

1 b (barn) = 10^{-24} cm^2

In a relativistic & quantum context, it seems natural to fix the two characteristic constants c and $\hbar$ to 1 in order to simplify the formulas and avoid errors.

$\hbar = 1 = J\ .\ s$, action has no units then $[t] = [E]^{-1} = \text{GeV}^{-1}$

$c = 1$, velocity has no units then $[d] = [t] = [E]^{-1} = \text{GeV}^{-1}$

The velocity of particle is then $v/c = \beta$ (between -1 and 1)

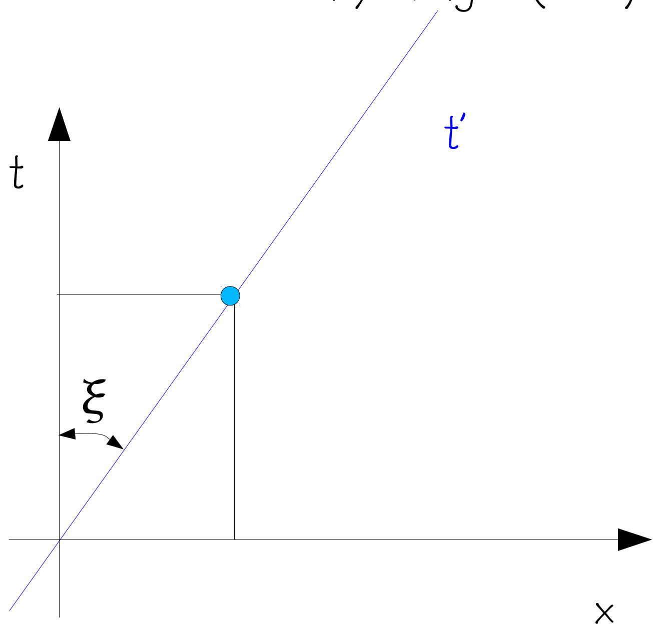
To come back to the MKS system, one needs to multiply or divide by $\hbar c$, c or $\hbar$ depending on the physics observable.

exercise : convert $G_N = 6.6742\ 10^{-11}$ m^3 Kg^{-1} s^{-2} into natural units.

Special relativity

What is important in relativity, this is what is absolute!

All particles move at the velocity of light (c = 1) but in space-time ! this is an absolute.



A free particle that propagates at c=1 along a universe line.

(0,t,x) is an inertial or Galilean frame or non-accelerated frame.

Generalized theorem of Pythagoras :

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu$$

In the Minkowski space, the space-time of special relativity :

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(t')^2 = t^2 - x^2 = t^2 - (\beta t)^2 = (1-\beta^2) t^2 \Rightarrow t = \gamma t' \text{ and } x = \beta \gamma t' \text{ with } \gamma = (1-\beta^2)^{-1/2}$$

Or : $t = \cosh(\xi) t'$ and $x = \sinh(\xi) t'$ with $\cosh(\xi) = \gamma$ and $\sinh(\xi) = \gamma \beta$

$$\beta = \tanh(\xi)$$

ξ is the rapidity, which is not an absolute !

For a massless particle, a photon, t=x then t' = 0 everywhere on its universe line

ξ is then infinite

Special relativity

Let us consider : ξ_1 and ξ_2 that describe in two different inertial frames the same particle,

$$\begin{pmatrix} t_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} \cosh(\xi_1) & \sinh(\xi_1) \\ \sinh(\xi_1) & \cosh(\xi_1) \end{pmatrix} \begin{pmatrix} t' \\ 0 \end{pmatrix} \qquad \begin{pmatrix} t_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} \cosh(\xi_2) & \sinh(\xi_2) \\ \sinh(\xi_2) & \cosh(\xi_2) \end{pmatrix} \begin{pmatrix} t' \\ 0 \end{pmatrix}$$

then : $$\begin{pmatrix} t_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} \cosh(\xi) & \sinh(\xi) \\ \sinh(\xi) & \cosh(\xi) \end{pmatrix} \begin{pmatrix} t_2 \\ x_2 \end{pmatrix} \quad \text{avec} \quad \xi = \xi_1 - \xi_2$$

This is the special Lorentz transformation (Lorentz boost) between (t1,x1) and (t2,x2)

Relative velocity between the inertial frames : $\beta_r = \tanh(\xi) = \dfrac{\beta_1 - \beta_2}{1 - \beta_1 \beta_2}$

if $\beta_1 = 1$ then $\beta_r = 1$ whatever the value of β_2 .

$$t' = \sqrt{t_1^2 - x_1^2} = \sqrt{t_2^2 - x_2^2}$$

is called the proper time. This is an absolute quantity !

The absolute motion is the combination of the motion in space and the motion in time. It always occurs at the velocity of light.

A photon always moves with $\beta = 1$, its rapidity is infinite and then $x = t$ regardless the inertial frame in which it is observed. Its proper time is always zero !

Special relativity

Contravariant 4-vectors : Q^{μ}

$x^{\mu}=(t,\vec{r})=(t,x,y,z)$ a space-time event

$P^{\mu}=(E,\vec{p})$ 4-momentum (E being the total energy)

$A^{\mu}=(V,\vec{A})$ electromagnetic potential

$J^{\mu}=(\rho,\vec{J})$ 4-current (charge density – charge current)

$\gamma^{\mu}=(\gamma^{0},\vec{\gamma})$ Dirac matrices

$\partial^{\mu}=(\frac{\partial}{\partial t},-\vec{\nabla})$ Derivation operators

Covariant 4-vectors : $Q_{\mu}=g_{\mu\nu}Q^{\nu}$

Lorentz invariance of the modulus and the scalar product of 4-vectors :

$$Q^{2}=Q^{0}-(\vec{Q})^{2}=Q_{\mu}Q^{\mu}=g_{\mu\nu}Q^{\mu}Q^{\nu}=cte$$

When moving from one inertial frame to another.

$$Q_{1}\cdot Q_{2}=(Q_{1})_{\mu}(Q_{2})^{\mu}=(Q_{1})_{0}(Q_{2})_{0}-\vec{Q}_{1}\vec{Q}_{2}=cte$$

Special relativity

Concerning the energy and the momentum :

The absolute energy (or the total energy observed at rest) which is called the mass of a particle is obtained through :

$$m^2 = P^2 = E^2 - p^2 \;\Rightarrow\; E^2 = p^2 + m^2$$

But as :

$$\begin{pmatrix} E \\ p \end{pmatrix} = \begin{pmatrix} \cosh(\xi) & \sinh(\xi) \\ \sinh(\xi) & \cosh(\xi) \end{pmatrix} \begin{pmatrix} m \\ 0 \end{pmatrix}$$

we obtain : $E = \gamma m$ $\qquad p = \gamma \beta m$

and $\beta = \dfrac{x}{t} = \dfrac{p}{E}$

The mass of particle is an absolute quantity.

A photon is a massless particle since it moves with $\beta=1$ in all inertial frames, then its energy and its momentum are identical.

Kinetic energy K : E = m + K , then : $K = m(\gamma - 1)$ and $p = \sqrt{K(K+2m)}$

reaction & decay (disintegration) kinematics

In relativity, the total momentum and total energy are always conserved regardless of the nature of the collision (elastic or inelastic scattering) or disintegration.
This results in the conservation of the total 4-momentum.

Likewise, in a given inertial frame, the 4-momentum modulus squared remains constant during the collision or the disintegration.

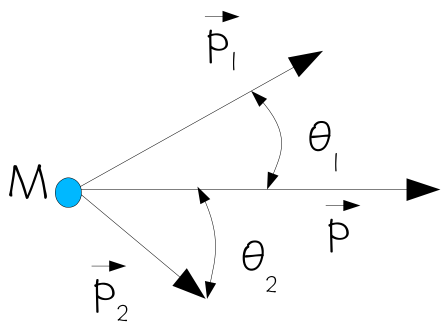
At a given time, any scalar product of two 4-momenta is invariant under a change of inertial frames.

Exercise : calculate the total energy of a muon emitted in a pion decay in the inertial frame bound to the pion, assuming that the neutrino emitted can be considered as a massless particle :

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Exercise

A neutral particle of mass M carrying a momentum p decays (disintegrates) into an e^+e^- pair. Find the expressions of p_1, p_2 et $\cos\theta_2$ if θ_1, the outgoing electron angle, is known. We will assume that $M \gg m_e$ and that consequently the electron and the positron can be treated as massless particles..



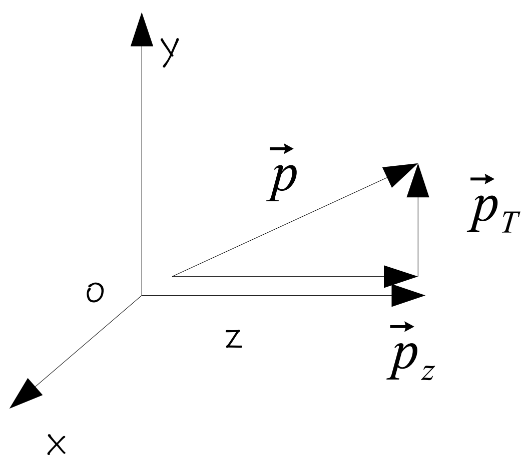
Analyze the solution for $\theta_1 = 0$ and for $\theta_1 = 90^0$.

Can we solve that problem if one does not know θ_1 ?

Show that the emission occurs with a non-zero opening angle ($\theta = \theta_1 + \theta_2$) and determine the expression of the minimal value.

Kinematics

The inertial frame R is endowed with a Cartesian coordinate system whose (0,z) axis is directed towards a particular direction, that of a particle beam that collides.
The transverse mass m_T of a particle of mass m (which could be produced in the collisions) is defined from the orthogonal-to-(O,z) projection of its momentum according to the following equation :



$$m_T^2 = m^2 + p_T^2 = m^2 + p_x^2 + p_y^2$$

$$m_T^2 = E^2 - p_z^2$$

$$P = (E = m_T \cosh \xi,\ p_T = \sqrt{p_x^2 + p_y^2},\ p_z = m_T \sinh \xi)$$

$$\xi = \tanh^{-1}(\beta_z) = \tanh^{-1}(p_z / E) = \ln\left(\frac{E + p_z}{m_T}\right) = \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right)$$

The particle rapidity ξ varies from $-\infty$ to $+\infty$

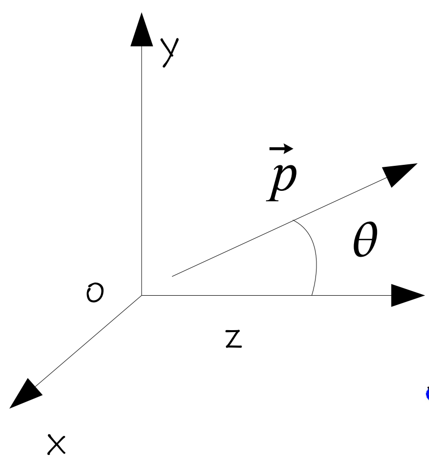
In another inertial frame R* that moves with respect to R along (o,z) with a constant velocity β_r, one can show that : $\xi^* = \xi - \xi_r$ and therefore : $d\xi^* = d\xi$

Kinematics

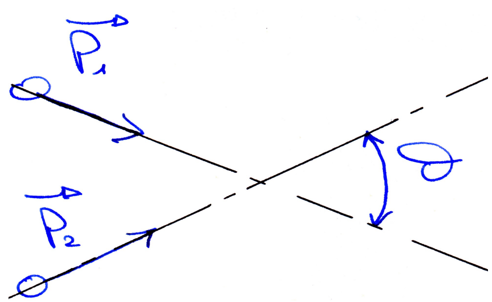
For a ultra-relativistic particle : $E \approx p$ *and* $m_T = E_T \approx p_T$

$$\xi = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \approx \frac{1}{2}\ln\left(\frac{p+p\cos\theta}{p-p\cos\theta}\right) = \frac{1}{2}\ln\left(\frac{\cos^2\theta/2}{\sin^2\theta/2}\right) = -\ln(\tan(\theta/2)) = \eta$$

η is called the pseudo-rapidity



$$\sinh\eta = \cot\theta \qquad \cosh\eta = 1/\sin\theta \qquad \tanh\eta = \cos\theta$$



Total energy in the center-of-mass frame of the reaction (where the threshold of the reaction is judged).

In the center-of-mass frame, the total momentum is zero.

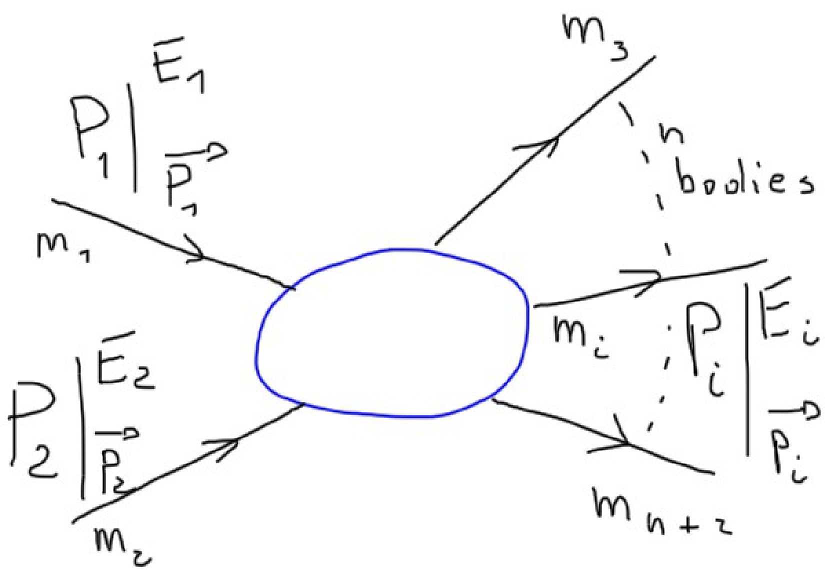
$$E_{cm} = \sqrt{s} = \sqrt{(P_1+P_2)^2} = \sqrt{(P_{1cm}+P_{2cm})^2} = \sqrt{(E_{1cm}+E_{2cm})^2}$$
$$= (m_1^2+m_2^2+2E_1E_2(1-\beta_1\beta_2\cos\theta))^{1/2}$$

On a collider, cosΘ=-1 , and in a ultra-relativistic mode : $E_{cm} = 2\sqrt{E_1 E_2}$

LHC (pp) : $E_{cm} = 14\,\text{TeV}$

Tevatron $(p\bar{p})$: $E_{cm} = 1.96\,\text{TeV}$ LEP $(e^- e^+)$: $E_{cm} = 209\,\text{GeV}$

Reaction cross section



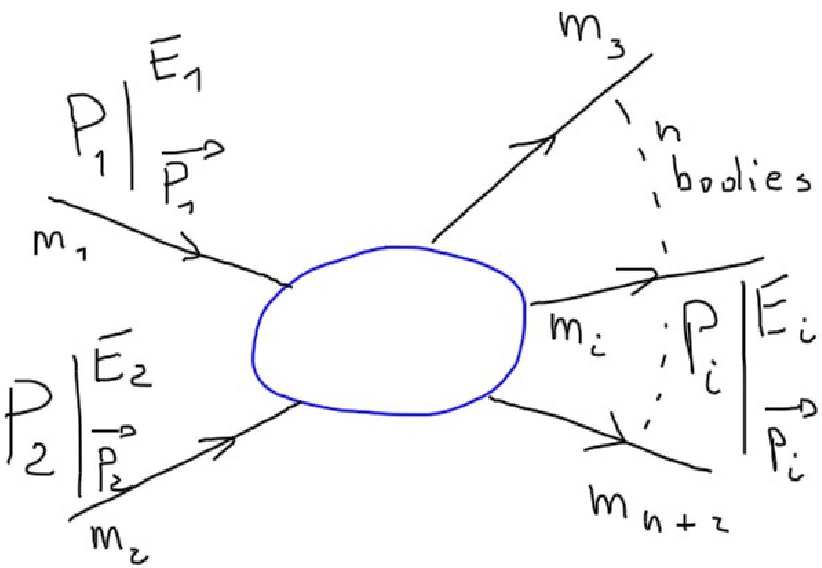
Two particles in the input channel
and n particles in the output channel.

The cross section is a physical quantity which has the dimension of a surface. It is related to the probability of two particles to undergo a given reaction. Geometrically, it corresponds to the same area disk, placed perpendicularly to the projectile propagation direction and centered on the target particle, when this one is observed at rest. In other words, it represents the apparent surface of the target for a given projectile.

By convention, the reaction cross section of two particles is defined in an inertial frame where one of them is at rest. For two specified particles, it only depends upon their relative velocity, which is a Lorentz invariant. It follows that the cross section defined this way is a relativistic invariant.

exercise : Show that the relative velocity of two particles is a relativistic invariant. Care should be taken to clearly define the relative velocity from the relativistic composition formula of velocities.

Reaction cross section



The reaction rate (1+2-> 3 +...) is given by :

$$\frac{dN(1+2\to 3+\ldots)}{dt}=\sigma(1+2\to 3+\ldots)\ L_{12}$$

(1+2 -> 3+...) reaction cross section

$$[\sigma]=\mathrm{m}^2$$

often in barn = 10^{-28} m^2

or GeV^{-2} in natural units.

The collider instantaneous luminosity. It expresses the number of particle 1 and 2 crossings per unit area and per second.

$$[L]=\mathrm{m}^{-2}\mathrm{s}^{-1}$$

often in cm^{-2} s^{-1}

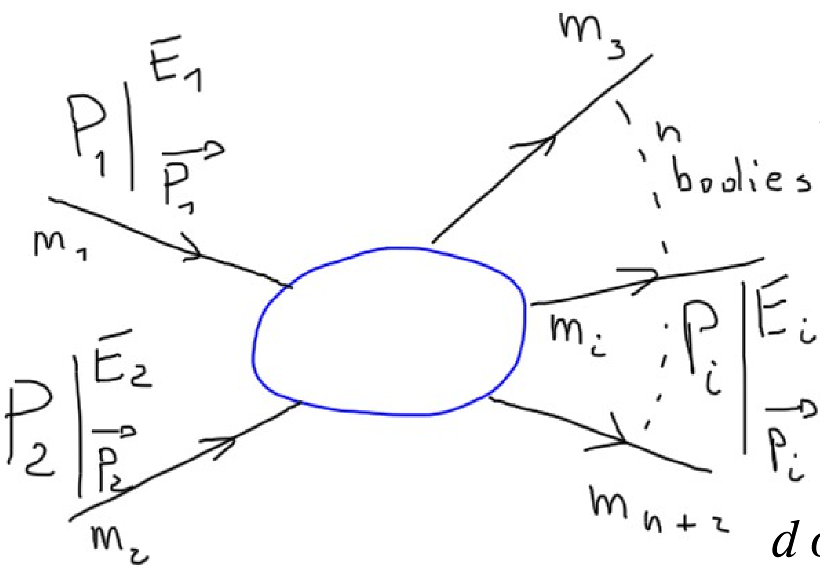
Example (LHC) : Nominal luminosity L_{pp} = 10^{34} cm^{-2} S^{-1}

$$\sigma(p+p\to W+X)=140\,\mathrm{nb}$$

called W boson inclusive production

$$\frac{dN}{dt}=1400\ \text{inclusive W}\ \mathrm{s}^{-1}$$

Reaction cross section



Differential cross section : production of n particles according to specified 4-momenta .

Lorentz invariant matrix element. Obtained applying the Feynman rules and the interaction theory.

$$d\sigma = \frac{(2\pi)^4 |M|^2}{4\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2}} \, d\Phi_n(P = P_1 + P_2 ; P_3, \ldots, P_{n+2})$$

3n dimensional Lorentz invariant phase space volume element. It quantifies the number of kinematically accessible states.

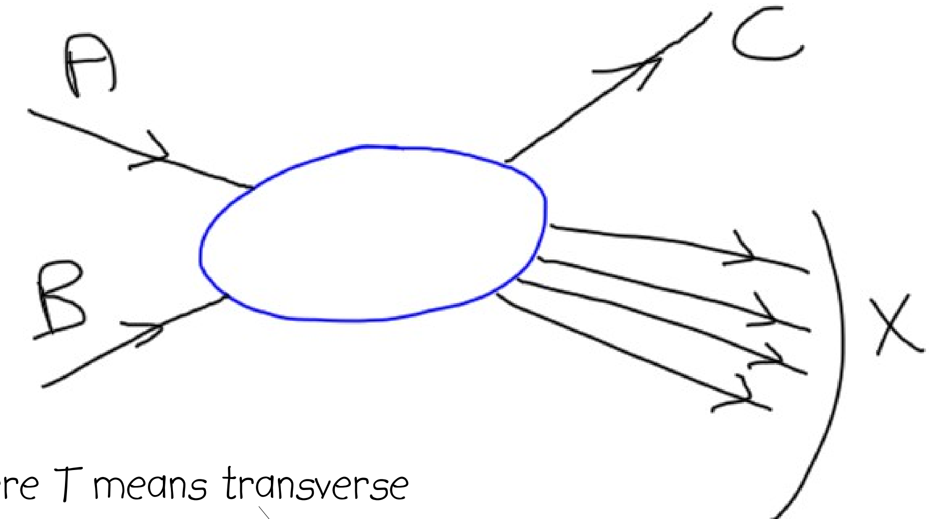
$$d\Phi_n(P ; P_3, \ldots, P_{n+2}) = \delta^4(P - \sum_{i=1}^{n} P_{i+2}) \prod_{i=3}^{n+2} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

where : $(P_i = (E_i, \vec{p}_i) ; i = 1, n+2)$

$\delta^4(P - \sum_{i=1}^{n} P_{i+2})$ enforces conservation of total energy and total momentum, in other words total 4-momentum.

with : $\sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2} = \sqrt{(P_{1\mathrm{cm}} \cdot P_{2\mathrm{cm}})^2 - m_1^2 m_2^2} = p_{cm} E_{cm}$ exercise : show that relation.

Inclusive reactions



If only the particle C of interest is detected in the outgoing channel :

Transverse momentum of C

$$E_C \frac{d^3\sigma}{d^3 p_C} = \frac{d^3\sigma}{d\Phi_C \; d\xi_C \; p_{C\,T} \; dp_{C\,T}}$$

Here T means transverse

$$d^3 p_C = p_{C\,T} \; dp_{C\,T} \; d\Phi_C \; dp_{C\,z}$$

with : $dp_{C\,z} = m_{C\,T} \cosh \xi_C \; d\xi_C = E_C \; d\xi_C$

rapidity of C (or pseudo-rapidity if C is ultra-relativistic)

Invariant differential cross section

C azimuthal angle

Φ_C, ξ_C, p_{CT} are the natural variables of inclusive reaction measurement on an ultra-relativistic collider.

$d\xi, p_T, dp_T, \Phi$ and $d\Phi$ are the same in all inertial frames moving along the (o, z) collision axis.

Relativistic Quantum Field Theory

A quantum field is a physical entity defined everywhere in space-time, that informs about the presence and the properties of the physical objects that populate it.
It allows to model a Universe containing an infinite number of objects.
It may also be seen as a network of quantum operators that evolve as a function of time and space.

In the Lagrangian formalism :

Lagrangian density of a field : $\varphi(x^\mu)$

$$L(\varphi(x^\mu),\partial_\mu\varphi(x^\mu))$$

Lagrangian of $\varphi(x^\mu)$ that depends only on time

$$L(t)=\int_V L(\varphi(x^\mu),\partial_\mu\varphi(x^\mu))\,d^3\vec{x}$$

Hamiltonian density :

$$H=\frac{\partial\varphi}{\partial t}\frac{\partial L}{\partial\dot{\varphi}}-L$$

Energy :

$$E=\int_V H\,d^3\vec{x}$$

The field equations of motion are obtained by requiring the stationarity of the action integral :

$$S=\int_{t_1}^{t_2} L(t)\,dt=\int_{t_1}^{t_2}\int_V L(\varphi(x^\mu),\partial_\mu\varphi(x^\mu))\,d^4x$$

$$\delta S=0\;\Rightarrow\;\frac{\partial L}{\partial\varphi}-\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\varphi)}\right)=0$$ Euler-Lagrange equations

Because of the space-time symmetry properties, the Lagrangian density must be invariant under any transformation of the restricted Poincaré group : translations in time and space, spatial rotations and Lorentz boosts. This leads to the conservation during the motion of the total energy, of the total momentum, of the total angular momentum and of the total 4-momentum modulus.

Selected example in non-quantum field theory : Maxwell equations

$A^{\mu}=(V,\vec{A})$ electromagnetic potential (electromagnetic field)

$J^{\mu}=(\rho,\vec{J})$ current source $\vec{E}=-\vec{\nabla}V-\frac{\partial\vec{A}}{\partial t}$ $\vec{B}=\overrightarrow{rot}\,\vec{A}$

$F^{\mu\nu}=\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}$ antisymmetric tensor field

$L=-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-J^{\mu}A_{\mu}$ Lagrangian density with an interaction term.

The properties of the antisymmetric tensor field and the Euler-Lagrange equations applied to this Lagrangian density lead to the covariant Maxwell equations.

$\partial_{\alpha}F_{\beta\gamma}+\partial_{\beta}F_{\gamma\alpha}+\partial_{\gamma}F_{\alpha\beta}=0$ structural equations

$\partial_{\alpha}F^{\alpha\beta}=J^{\beta}$ contextual equations

exercise : find the covariant Maxwell equations
- by exploiting the properties of the antisymmetric tensor field
- by applying the Euler-Lagrange equations to the above Lagrangian density

Real scalar hermitian field of free spin 0 particles

Can be used to model a neutral pion : π^0 $\varphi = \varphi^+$

Lagrangian density : $L = \frac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2)$

Kinetic energy term

Mass term

Field equation : $(\partial_\mu \partial^\mu + m^2)\varphi = 0$ Klein-Gordon equation

exercise : show that equation.

which may also be obtained by applying the correspondence principle to the relativistic equation :

$$E^2 = \vec{p}^2 + m^2$$

$$E \rightarrow i\partial_0 \qquad \vec{p} \rightarrow -i\vec{\nabla}$$

Field operator :

plane wave evolution

$$\varphi(x^\mu) = \sum_{\vec{k}} \frac{1}{\sqrt{(2\omega)}} (a_{\vec{k}} e^{-ik\cdot x} + a_{\vec{k}}^+ e^{+ik\cdot x}) \qquad k^0 = \omega = \sqrt{\vec{k}^2 + m^2}$$

particle creation operator

particle annihilation operator

Non-hermitian free field of spin 0 particles

Can be used to model charged pions : π^+, π^- then : $\varphi \neq \varphi^+$

Lagrangian density : $$L = \frac{1}{2}\left(\partial_\mu \varphi \, \partial^\mu \varphi^+ - m^2 \varphi \varphi^+\right)$$

Kinetic energy term

Mass term

Field equations : $$(\partial_\mu \partial^\mu + m^2)\varphi = (\partial_\mu \partial^\mu + m^2)\varphi^+ = 0$$ Klein-Gordon equations

Field operator :

$$\varphi(x^\mu) = \sum_{\vec{k}} \frac{1}{\sqrt{(2\omega)}}\left(a_{\vec{k}} e^{-ik\cdot x} + b^+_{\vec{k}} e^{+ik\cdot x}\right)$$

$$k^0 = \omega = \sqrt{\vec{k}^2 + m^2}$$

Positively-charged particle annihilation operator

Negatively-charged particle creation operator

Free field of spin 1/2 particles

Can be used to model the matter particles : leptons and quarks

Dirac free field with 4 complex and non-hermitian components : $\psi^{\alpha}(x^{\mu})$

$\alpha = 1,2,3,4$

Spinorial index, not to confuse with the index in the Minkowski space

Lagrangian Dirac density : $L = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m)\psi$

Kinetic energy term

Mass term

Minkowski space index

γ^{μ} : 4 Dirac 4-dimensional matrices

with $\bar{\psi} = \psi^{+}\gamma^{0}$

Field equations : $(i\gamma_{\mu}\partial^{\mu} - m)\psi = 0$

Dirac equations

4-component Dirac spinors

Field : $\varphi(x^{\mu}) = \sum_{\vec{k}} \sum_{r=1,2} \sqrt{\frac{m}{\omega}} (a_{\vec{k}r} u_r(\vec{k}) e^{-ik\cdot x} + b^{+}_{\vec{k}r} v_r(\vec{k}) e^{+ik\cdot x})$ $\quad k^0 = \omega = \sqrt{\vec{k}^2 + m^2}$

indice de spin

annihilitation operator of a positively-charged particle of mass m.

creation operator of a negatively-charged particle of mass m

Dirac matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad (\gamma^0)^+ = \gamma^0 \qquad (\vec{\gamma})^+ = -\vec{\gamma}$$

Dirac representation of Dirac matrices :

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \ \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \ , \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

σ_i are the 2 × 2 Pauli matrices $\qquad (\sigma_i)^+ = \sigma_i$

$$\vec{\alpha} = \gamma^0\vec{\gamma} = (\vec{\alpha})^+ \qquad \vec{\Sigma} = \gamma^5\vec{\alpha} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \qquad \hat{\Lambda} = \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}$$

is the helicity operator (projection of spin on the propagation direction)

$$(\hat{\Lambda})^2 = \left(\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\right)^2 = 1$$

The helicity is an *observable* whose eigenvalues are + 1 and -1 for fermions.
Fermions may be labelled by their helicity..

$$(1-\gamma^5)^2 = 2\,(1-\gamma^5) \qquad \{\gamma^\mu, \gamma^5\} = 0$$

$$\gamma^5(1-\gamma^5) = -\,(1-\gamma^5)$$

Dirac spinors

spin ½ and - ½

particles

$$u(\vec{p},s)=\sqrt{\frac{E+m}{2\mathrm{m}}}\begin{pmatrix}\chi_s\\ \frac{\vec{\sigma}.\vec{p}}{E+m}\chi_s\end{pmatrix}$$

antiparticles

$$v(\vec{p},s)=\sqrt{\frac{E+m}{2E}}\begin{pmatrix}\frac{\vec{\sigma}.\vec{p}}{E+m}\chi_s\\ \chi_s\end{pmatrix}$$

$$\chi_+=\begin{pmatrix}1\\0\end{pmatrix},\ \chi_-=\begin{pmatrix}0\\1\end{pmatrix}$$

$$\bar{u}(\vec{p},s)\,u(\vec{p},r)=\delta_{rs}\qquad \bar{v}(\vec{p},s)\,v(\vec{p},r)=-\delta_{rs}\qquad \bar{v}(\vec{p},s)\,u(\vec{p},r)=0$$

Notation : $\not{a}=\gamma^{\mu}a_{\mu}$ then $(\not{p}-m)u(\vec{p},s)=0$ and $(\not{p}+m)v(\vec{p},s)=0$

When the (o,z) spin quantization axis is taken as the propagation axis : $\hat{z}\parallel\vec{p}$

$$u(\vec{p},\lambda=1)=\sqrt{\frac{E+m}{2\mathrm{m}}}\begin{pmatrix}1\\0\\ \frac{p}{(E+m)}\\0\end{pmatrix}\qquad u(\vec{p},\lambda=-1)=\sqrt{\frac{E+m}{2\mathrm{m}}}\begin{pmatrix}0\\1\\0\\ \frac{-p}{(E+m)}\end{pmatrix}$$

$$v(\vec{p},\lambda=1)=\sqrt{\frac{E+m}{2\mathrm{m}}}\begin{pmatrix}\frac{p}{(E+m)}\\0\\1\\0\end{pmatrix}\qquad v(\vec{p},\lambda=-1)=\sqrt{\frac{E+m}{2\mathrm{m}}}\begin{pmatrix}0\\ \frac{-p}{(E+m)}\\0\\1\end{pmatrix}$$

exercise :

Show that : $(\not{p}-m)u(\vec{p},s)=0$ can be written as : $H\ u(\vec{p},s)=E\ u(\vec{p},s)$

with : $H=\vec{\alpha}\cdot\vec{p}+\gamma^0 m$

And consequently : $H^2=(\vec{p})^2+m^2=E^2$

which was the objective of Dirac, i.e. : to establish some first order relativistic quantum equations.
The surprise was to discover that these equations had 4 independent solutions : a particle and its antiparticle, each with two spin states.

Free field of massive and neutral spin 1 particles

Proca Lagrangian density : $L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$ with : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

antisymmetric tensor field

Kinetic energy

mass term

A^μ is a Minkowski 4-vector field, a priori possessing 4 independent components.

For massive particles (Proca field), the field equations are :

$\partial_\mu A^\mu = 0$ which implies that the 4 components of the field are not independent.

$(\partial_\mu \partial^\mu + m^2) A^\nu = 0$ each component is a spin 0 field which is solution of the Klein Gordon equation.

This will be used to transform a massless vector field into a massive field.

Free field of massive and neutral spin 1 particles

Field : $A^\mu = (A^0, \vec{A})$ in a basis where (o,z) – spin quantification axis - is // to $\vec{k}$

$$\vec{A}(x^\mu) = \sum_{\vec{k}} \frac{1}{\sqrt{2\,\omega}} \left\{ \left(\hat{k}\frac{\omega}{m} a_{\vec{k}L} + \sum_{t=1}^{2} \vec{\epsilon}_{\vec{k}}^{\,T} a_{\vec{k}T} \right) e^{-ik\cdot x} + \text{H. c.} \right\}$$

annihilation operator of a longitudinal boson of zero helicity.

annihilation operator of a transverse boson of +1 or -1 helicity.

$$\omega = k^0 = \sqrt{\vec{k}^2 + m^2}$$

$$A^0(x^\mu) = \sum_{\vec{k}} \frac{1}{\sqrt{2\,\omega}} \left\{ \frac{|\vec{k}|}{m} a_{\vec{k}L} e^{-ik\cdot x} + \text{H. c.} \right\}$$

$$\epsilon_{\vec{k}}^{1} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon_{\vec{k}}^{2} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \hat{k} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

polarisation vectors of the associated wave

Free field of massless and neutral spin 1 particles

Lagrangian density :

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

with : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

antisymmetric tensor field

Kinetic energy term

A^μ is a Minkowski 4-vector field but only two of its components are independent :

-1 and +1 helicity states

Field equations : $\partial_\alpha F^{\alpha\beta} = 0$

Field : in the Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$

$$\vec{A}(x^\mu) = \sum_{\vec{k}} \frac{1}{\sqrt{2\,\omega}} \left\{ \sum_{t=1}^{2} \vec{\epsilon}_{\vec{k}}^{\,T} a_{\vec{k}T} e^{-ik\cdot x} + \text{H. c.} \right\}$$

Interacting fields : local gauge theories

Quantum Electrodynamics QED .

Let's consider the Dirac Lagrangian density of a free electron : $L_e = \bar{\psi}(x^\nu)(i\gamma^\mu \partial_\mu - m)\psi(x^\nu)$

This expression is invariant under any of these transformations : $\bar{\psi}' = e^{i\epsilon}\psi$
i.e. is not sensitive to the choice of the global phase of the field

One may try to further explore this phase invariance by requiring that the phase choice could be different in distinct points of the Minkowski space :

$$\bar{\psi}'(x^\nu) = e^{iq\lambda(x^\nu)}\psi(x^\nu)$$ local gauge (phase) transformation

where q is real and $\lambda(x^\nu)$ is an arbitrary real differentiable function.

This is only possible if we replace the simple derivatives by covariant derivatives :

$D_\mu = \partial_\mu + iq A_\mu(x^\nu)$ where $A_\mu(x^\nu)$ is a 4-vector massless field (called gauge) that transforms according to the following expression :

$$A'_\mu(x^\nu) = A_\mu(x^\nu) - \partial_\mu \lambda(x^\nu)$$

exercise : Show that

$$L_e^{inv} = \bar{\psi}(x^\nu)(i\gamma^\mu D_\mu - m)\psi(x^\nu)$$

is invariant under a local phase transformation

Local gauge theories

The QED invariant Lagrangian density may be written as :

$$L_e^{inv} = \bar{\psi}\,(\,i\,\gamma^\mu \partial_\mu - m\,)\,\psi - q\,\bar{\psi}\,\gamma^\mu \psi\;A_\mu$$

to which one needs to add the Kinetic energy term of A_μ which is a local gauge invariant.

After defining : $j^\mu = \bar{\psi}\,\gamma^\mu \psi$, the final QED Lagrangian density reads :

$$L_e^{QED} = \bar{\psi}\,(\,i\,\gamma^\mu \partial_\mu - m\,)\,\psi - q\,j^\mu\;A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

free electron field

interaction term

free gauge field

j^μ is the electron current, q being the electron charge.

However the gauge field, the photon must remain massless as :

$$\frac{m^2}{2}A_\mu A^\mu$$ is not a local gauge invariant.

exercise : show that the mass term of a photon is not locally gauge invariant.

The photon must be a massless particle to preserve the local gauge invariance.

One may also notice that the electron charge – which determines its coupling to the photon – is intimately related to freedom of local gauge (phase) choice of the field.

Local gauge theories

Field equations : $\partial_\nu F^{\nu\mu} = q\, j^\mu$ Contextual Maxwell equations

However : $\partial_\mu \partial_\nu F^{\nu\mu} = 0 \;\Rightarrow\; \partial_\mu j^\mu = 0$ then j^μ is a conserved current and the volume integral of its time component is a constant of motion.

The total charge is conserved !

exercise : show that the volume integral of the charge is a constant of motion.

Summary : by requiring the local gauge invariance through a transformation of the U(1) group, one obtains :

- a relativistic quantum theory which describes the interaction between photons and electrons (but also of positrons) :
- the Maxwell equations ;
- a principle that forces the photon to remain massless ;
- the conservation of the electron current ;
- the conservation of the charge .

The standard model

We have seen that a conserved current is related to an internal symmetry acting locally on the field phase : local gauge transformation.

But at the beginning of these lectures we saw that there exist :

- a general invariance of the U(1) type (e -> e , quark -> quark ...) ;
- a general weak isospin invariance of the SU(2) type ; (e <-> neutrino ...)
- a general color invariance of quarks of the SU(3) type ; (q_{red} <-> q_{blue} ...)

U(1) symmetry $\psi' = e^{ig_1 \frac{Y}{2} \lambda(x^\mu)} \psi$ Y is the weak hypercharge

SU(2) symmetry $\begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix}_L' = e^{i g_2 \vec{\alpha}(x^\mu) . \frac{\vec{\tau}}{2}} \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix}_L$ 3 matrices $\vec{\tau}$ Pauli matrices

SU(3) symmetry $\begin{pmatrix} q_r' \\ q_v' \\ q_b' \end{pmatrix} = e^{i g_3 \vec{a}(x^\mu) \cdot \frac{\vec{\lambda}}{2}} \begin{pmatrix} q_r \\ q_v \\ q_b \end{pmatrix}$ 8 matrices $\vec{\lambda}$ Gell-Mann matrices.

Standard model : the free fields

Experimentally, ultra-relativistic electrons and neutrinos produced in beta decays (weak interaction) are always emitted in a negative (-1) helicity state (parity violation).
For ultra-relativistic particles, the negative helicity projector reads :

$$\frac{1}{2}\left(1-\frac{\vec{\Sigma}\,.\,\vec{p}}{|\vec{p}|}\right)=\frac{1}{2}(1-\hat{\Lambda})\simeq\frac{1}{2}(1-\gamma^5)$$

which is the left-handed chirality projector

But contrary to helicity projectors, chirality projectors are Lorentz invariants and consequently they are used instead of the helicity projectors to build the free fields in the Lagrangian density.

As experimentally, the weak interaction seems to apply only to the left-handed chiral fields :

$$L_e=\begin{pmatrix}\Psi_{\nu_e}\\ \Psi_e\end{pmatrix}_L=\frac{1}{2}(1-\gamma^5)\begin{pmatrix}\Psi_{\nu_e}\\ \Psi_e\end{pmatrix}$$

isospin : T=1/2

same thing for : μ and τ

$$L_1=\begin{pmatrix}\Psi_u\\ \Psi_d\end{pmatrix}_L=\frac{1}{2}(1-\gamma^5)\begin{pmatrix}\Psi_u\\ \Psi_d\end{pmatrix}$$

same thing for the two other quark families.

Standard model : the free fields

The remaining parts (right-handed chiral fields) are isospin singlets T=0

$$R_e = \frac{1}{2}(1+\gamma^5)\psi_e \ , \ R_u = \frac{1}{2}(1+\gamma^5)\psi_u \ , R_d \ldots.$$

Color is an additional index which is used to designate the quarks fields $L_1 = \begin{pmatrix} \psi_{u_\alpha} \\ \psi_{d_\alpha} \end{pmatrix}$ avec $\alpha = r, b, g$

Standard model : interaction of fermions

Complete Lagrangian density of free fields and their interactions :

$$L^{SM}_{fermions} = \sum_{F=L_l, L_i, R_f} \bar{F}\, i\gamma^\mu D_\mu\, F$$

using the covariant derivative : $D_\mu = \partial_\mu + i g_1 \frac{Y}{2} B_\mu + i g_2 \frac{\vec{\tau}}{2} \vec{W}_\mu + i g_3 \frac{\vec{\lambda}}{2} \vec{G}_\mu$

where :

B_μ is the (spin 1) gauge boson of weak hypercharge interaction

W^i_μ are the three (spin 1) gauge bosons of weak isospin interaction

G^i_μ are the eight (spin 1) gauge bosons (gluons) of strong color interaction

Until now, all these particles are massless ! as :

-the mass terms of the gauge bosons are not locally gauge invariant (already seen p. 37)

-concerning the fermions : $m\bar{\psi}\psi = m\bar{\psi}\left(\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5)\right)\psi = m\bar{\psi}_R\, \psi_L + m\bar{\psi}_L\, \psi_R$

which is not a local gauge invariant.

U(1) SU(2)

Feynman Graphs & Rules

Mnemo-graphic technique which helps to provide the expression of invariant scattering matrix elements. This technique can be totally established within the covariant quantum theory of perturbation.
(TD Lee, Particle Physics and Introduction to Field Theory p 62).
(S. Weinberg, The Quantum Theory of Fields, Vol I , p. 259).

Lagrangian density of two interacting fields :

$$L = L_1 + L_2 + L_{12}$$

interaction density

free field densities

Interaction Hamiltonian density :

Usually the interaction Lagrangian density contains no time derivatives of the fields, then :

$$H = H_1 + H_2 + H_{12}$$

with :

$$H_{12} = -L_{12}$$

Interaction Hamiltonian :

$$\mathrm{H}_I = \int_V H_{12} d^3\vec{x}$$

Feynman Graphs & Rules : scattering matrix (S matrix)

The time evolution of a physical state is obtained by :

$$|\psi(t)> = U(t,t_0)\ |\psi(t_0)>$$

where : $U(t,t_0)$ is the time evolution operator from t_0 to t.

In classical quantum mechanics, this operator is represented by the Green function.

The S scattering matrix is then defined as : $S = \lim_{\substack{t_0 \to -\infty \\ t \to \infty}} U(t,t_0)$

In the covariant quantum theory of perturbations, one can show that :

$$U(t,t_0) = 1 - i\int_{t_o}^{t} dt_1 \mathrm{H}_I(t_1) + (-i)^2 \int_{t_o}^{t} dt_1 \int_{t_o}^{t_1} dt_2 \mathrm{H}_I(t_1)\mathrm{H}_I(t_2) + \ldots$$

first order

second order

The scattering probability amplitude from |i > to |f > is given by :

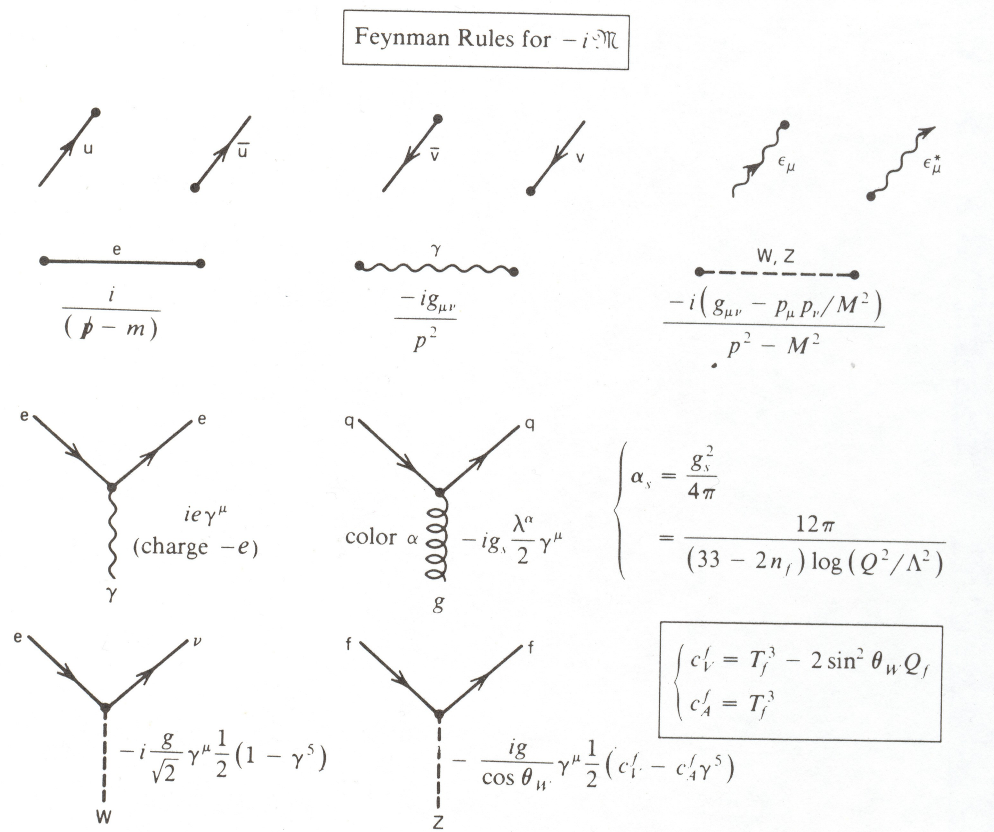
$$<f|S|i> = S_{fi} = \delta_{fi} + (2\pi)^4 \delta^4(P_f - P_i)\frac{(-iM_{fi})}{N}$$

Invariant matrix element that enters the calculation of the cross section

Total four-vector conservation

Normalisation factor

Feynman graphs and rules



f	Q_f	$(T_f^3)_L$	$(T_f^3)_R$
u, c, t	$+\frac{2}{3}$	$\frac{1}{2}$	0
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	0
ν_e, ν_μ, ν_τ	0	$\frac{1}{2}$	—
e, μ, τ	-1	$-\frac{1}{2}$	0

$\sin^2\theta_W \simeq 0.23$, $g\sin\theta_W = e$, $G = \frac{\sqrt{2}g^2}{8M_W^2} \simeq 1.17 \times 10^{-5}\ \text{GeV}^{-2}$

Phenomenology of Standard Model

The interaction terms induced by the weak hypercharge U(1) symmetry :

Examined for the first lepton family (similar for each family) :

here the hypercharge depends upon the lepton species and its chirality (like an electric charge depends upon the particle species ...)

$$L^e_{ferm}(U(1)) = -\bar{R}_e \gamma^\mu \left(g_1 \frac{Y_R}{2} B_\mu\right) R_e + \bar{L}_e \gamma^\mu \left(g_1 \frac{Y_L}{2}\right) B_\mu L_e$$

$$= -\frac{g_1}{2}\left[Y_R(\bar{R}_e \gamma^\mu R_e) + Y_L(\bar{L}_e \gamma^\mu L_e)\right] B_\mu$$

$$\bar{L}_e \gamma^\mu L_e = (\bar{\psi}_{\nu_{eL}}, \bar{\psi}_{eL}) \gamma^\mu \begin{pmatrix} \psi_{\nu_e L} \\ \psi_{e L} \end{pmatrix} = \bar{\psi}_{\nu_e L} \gamma^\mu \psi_{\nu_e L} + \bar{\psi}_{e L} \gamma^\mu \psi_{e L}$$

$$L^e_{ferm}(U(1)) = -\frac{g_1}{2}\left[Y_R(\bar{\psi}_{eR} \gamma^\mu \psi_{eR}) + Y_L(\bar{\psi}_{\nu_e L} \gamma^\mu \psi_{\nu_e L} + \bar{\psi}_{eL} \gamma^\mu \psi_{eL})\right] B_\mu$$

leptonic currents

B gauge boson

Phenomenology of Standard Model

Terms induced by the weak isospin SU(2) symmetry :

$$L^e_{ferm}(SU(2)) = \bar{L}_e \, i\gamma^\mu \, [i g_2 \frac{\vec{\tau}}{2}] \vec{W}_\mu \, L_e$$

$$L^e_{ferm}(SU(2)) = -\frac{g_2}{2} \, (\bar{\psi}_{\nu_{eL}}, \bar{\psi}_{eL}) \;\; \gamma^\mu \begin{pmatrix} W^3_\mu & W^1_\mu - i W^2_\mu \\ W^1_\mu + i W^2 & -W^3_\mu \end{pmatrix} \begin{pmatrix} \psi_{\nu_e L} \\ \psi_{e L} \end{pmatrix}$$

The charged weak bosons are defined by :

$$W^+ = (W^1_\mu - i W^2_\mu)/\sqrt{2} \qquad\qquad W^- = (W^1_\mu + i W^2_\mu)/\sqrt{2}$$

The Lagrangian then reads :

neutrino neutral current

leptonic charged weak current

$$L^e_{ferm}(SU(2)) = -\frac{g_2}{2}[\bar{\psi}_{\nu_e L} \gamma^\mu \psi_{\nu_e L} W^3_\mu + \sqrt{2}\bar{\psi}_{\nu_e L} \gamma^\mu \psi_{e L} W^+_\mu$$
$$+ \sqrt{2}\bar{\psi}_{e L} \gamma^\mu \psi_{\nu_e L} W^-_\mu - \bar{\psi}_{e L} \gamma^\mu \psi_{e L} W^3_\mu]$$

leptonic charged weak current

electron neutral current

Phenomenology of Standard Model

We notice that neutrinos interact as neutral currents with both gauge bosons :

$$[-\frac{g_1}{2} Y_L B_\mu - \frac{g_2}{2} W_\mu^3]\, \bar{\Psi}_{\nu_e L} \gamma^\mu \Psi_{\nu_e L}$$

hence the idea of introducing two new and orthogonal gauge bosons of physical interaction :

$$A_\mu = \frac{g_2 B_\mu - g_1 Y_L W_\mu^3}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad B_\mu = \frac{g_2 A_\mu + g_1 Y_L Z_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

$$Z_\mu = \frac{g_1 Y_L B_\mu + g_2 W_\mu^3}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad W_\mu^3 = \frac{-g_1 Y_L A_\mu + g_2 Z_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

For A_μ to be interpreted as the photon field , we would need that :

$$(-\frac{g_1}{2} Y_R B_\mu)\, \bar{\Psi}_{eR} \gamma^\mu \Psi_{eR} + (-\frac{g_1}{2} Y_L B_\mu + \frac{g_2}{2} W_\mu^3)\, \bar{\Psi}_{eL} \gamma^\mu \Psi_{eL}$$

contains : $-A_\mu(-e\, \bar{\Psi}_{eR} \gamma^\mu \Psi_{eR} - e\, \bar{\Psi}_{eL} \gamma^\mu \Psi_{eL}) = e\, \bar{\Psi}_e \gamma^\mu \Psi_e A_\mu$

Phenomenology of Standard Model

It's possible if :

$$e = \frac{g_1 g_2}{\sqrt{g_2^2 + g_1^2}} \qquad Y_L = -1 \qquad Y_R = 2Y_L = -2$$

$$A_\mu = \cos\theta_w B_\mu + \sin\theta_w W_\mu^3 \qquad Z_\mu = -\sin\theta_w B_\mu + \cos\theta_w W_\mu^3$$

$$\sin\theta_w = \frac{g_1}{\sqrt{g_2^2 + g_1^2}} \qquad \cos\theta_w = \frac{g_2}{\sqrt{g_2^2 + g_1^2}}$$

In this scheme, the unification of the weak hypercharge and weak isospin interactions takes place leading to the physical electromagnetic and weak neutral interactions.

$$g_2 = \frac{e}{\sin\theta_w} \qquad g_1 = \frac{e}{\cos\theta_w}$$

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137} \qquad \alpha_1 = \frac{g_1^2}{4\pi} \simeq \frac{1}{105} \qquad \alpha_2 = \frac{g_2^2}{4\pi} \simeq \frac{1}{32}$$

Both the weak hypercharge and weak isospin interactions feature coupling constants that are stronger than the electromagnetic interaction constant !

Electroweak interaction Lagrangian of fermions

leptonic charged current
weak interaction

$$L_{LB,l} = \frac{g_2}{2\sqrt{2}} \sum_{l=e,\mu,\tau} [\bar{\psi}_l \gamma^\mu (1-\gamma^5) \psi_{\nu_l} W^-_\mu + \bar{\psi}_{\nu_l} \gamma^\mu (1-\gamma^5) \psi_l W^+_\mu]$$

$$+ \frac{g_2}{2\sqrt{2}} \sum_{q=u,c,t} \sum_{q'=d,s,b} V_{qq'} [\bar{\psi}_{q'} \gamma^\mu (1-\gamma^5) \psi_q W^-_\mu + \bar{\psi}_q \gamma^\mu (1-\gamma^5) \psi_{q'} W^+_\mu]$$

$$+ \frac{g_2}{2\cos\theta} \sum_k [\bar{\psi}_k \gamma^\mu (T_3 - 2Q_k \sin^2\theta_w - T_3 \gamma^5) \psi_k] Z_\mu$$

$$+ e \sum_k Q_k \bar{\psi}_k \gamma^\mu \psi_k A_\mu$$

where : Q_k is the charge number

hadronic charged current
weak interaction

weak neutral current

electromagnetic current

$$Q = T_3 + \frac{Y}{2}$$

The electrical charge is made of weak hypercharge and weak isospin !

But where are the particle mass terms ?

There's none for the time being as they do not respect local gauge invariance.

The idea is to dynamically generate them through an appropriate interaction with vacuum or in other words the Universe ground state.

To achieve this goal, new non hermitian scalar fields (called Higgs Fields) are introduced. They form a new weak isospin doublet :

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix},\ |\Phi|^2 = |\Phi^+|^2 + |\Phi^0|^2,\ T = 1/2 \text{ and } Y = 1 \text{(extracted from the Glashow formula)}$$

$$\Phi^+ = \frac{\Phi_1 + i\,\Phi_2}{\sqrt{2}}$$ charged spin 0 boson

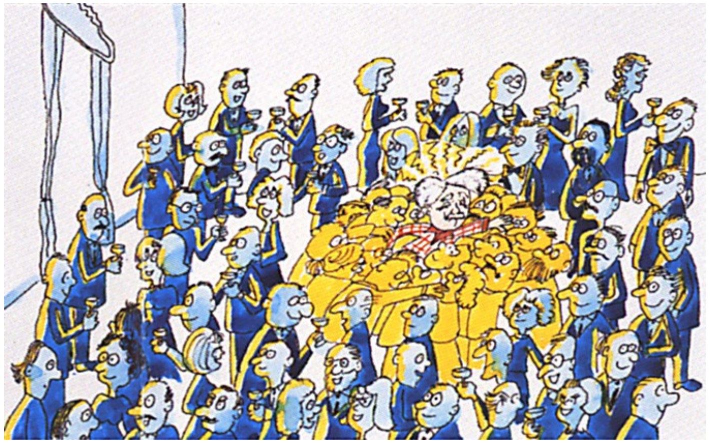
$$\Phi^0 = \frac{\Phi_3 + i\,\Phi_4}{\sqrt{2}}$$ neutral spin 0 boson

Doing so, 4 new degrees of freedom are introduced in the theory.

The mass of elementary particles

A new field (Higgs field) is present everywhere in the space vacuum.

An elementary particle propagating in vacuum is immersed in that field.



Through interaction, the Higgs field clusters around this particle, inducing a potential energy that takes the form of a mass term.

The Higgs, Englert and Brout mechanism

Kinetic energy term of Higgs fields with usage of covariant derivative :

$$|D_\mu \Phi|^2 = |(\partial_\mu + i\frac{g_2}{2}\vec{\tau}.\vec{W}_\mu + i\frac{g_1}{2}\hat{Y}B_\mu)\Phi|^2$$

Novelty : a potential energy term resulting from the auto-interaction of the Higgs Fields is introduced .

$$U(\Phi) = -\mu^2|\Phi|^2 + h|\Phi|^4 \qquad \text{with :} \qquad \mu^2 > 0$$

An important thing is to note that the neutral component of the Higgs fields does not couple to the photon :

$$(\frac{g_2}{2}\vec{\tau}.\vec{W}_\mu + \frac{g_1}{2}\hat{Y}B_\mu)\Phi = \begin{pmatrix} eA_\mu + \frac{g_2}{2}\frac{\cos 2\theta_w}{\cos\theta_w}Z_\mu & g_2\sqrt{2}W_\mu^+ \\ g_2\sqrt{2}W_\mu^- & -\frac{g_2}{2\cos\theta_w}Z_\mu \end{pmatrix}\Phi$$

In this expression, the gauge bosons have been replaced by the physical interaction bosons.

The Higgs, Englert and Brout mechanism

The Higgs field doublet may also be written as a isospin rotation of a particular state :

$$\Phi = \frac{1}{\sqrt{2}}(\lambda + \chi(x))\, e^{\frac{i}{\lambda}\vec{\theta}(x).\vec{T}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

But this exponential may be eliminated by applying a particular SU(2) gauge transformation called the unitary gauge transformation.

$$\Phi \xrightarrow{\text{unitary gauge}} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \lambda + \chi(x) \end{pmatrix}$$

Three degrees of freedom have disappeared. In fact, they become the missing longitudinal polarization components of the weak interaction bosons.

Let's have a look at the ground state of the Higgs field that minimizes the vacuum energy :

$$<0|\Phi|0> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \lambda + <0|\chi(x)|0> \end{pmatrix} = \frac{\lambda}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

as : $\chi(x)$

represent the quantum fluctuations around the minimum.

The Higgs, Englert and Brout mechanism

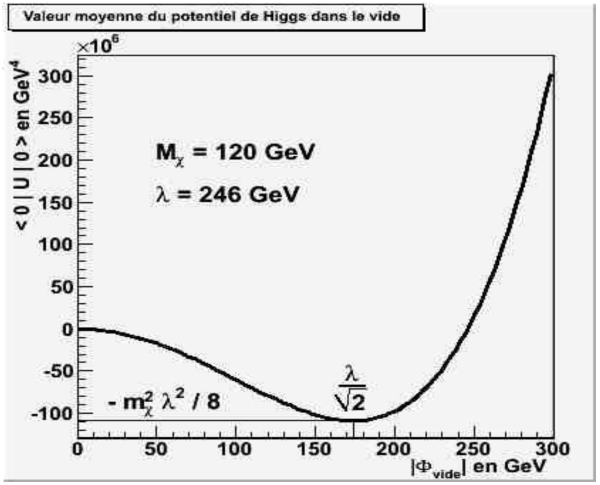
In the unitary gauge, we may rewrite the Higgs doublet as :

$$\Phi = \Phi_{vacuum} + \begin{pmatrix} 0 \\ \dfrac{\chi(x)}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad \Phi_{vacuum} = \frac{\lambda}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

If λ is a constant, then the associated kinetic energy is zero (field derivative)

Let's compute the average value of the potential energy induced by the Higgs field auto-interaction in vacuum :

$$<0|U(\Phi_{vacuum})|0> = -\mu^2 |\Phi_{vacuum}|^2 + h|\Phi_{vacuum}|^4 = -\frac{\mu^2}{2}\lambda^2 + \frac{h}{4}\lambda^4 \equiv V(\lambda)$$



This potential energy shows a local unstable minimum at : $\lambda = 0$

and a second stable minimum at :

$$\lambda^2 = \frac{\mu^2}{h}$$

which corresponds to a non zero expectation value of the Higgs field.

$$\Phi_{vacuum} = \frac{\mu}{\sqrt{2\,h}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

In other words to minimize the total vacuum energy, the Higgs field develops through auto-interaction a non zero expectation value which is constant in the Universe.

The Higgs, Englert and Brout mechanism

$$\Phi_{vacuum} = \frac{\mu}{\sqrt{2\,h}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The vacuum expectation value is not gauge invariant. This corresponds to a spontaneous symmetry breaking of SU(2)xU(1) -> U(1)

There remains a real scalar field of spin 0 : $\chi(x)$ which is the boson of Higgs, Englert and Brout (usually called the Higgs boson) which has been sought for more than 40 years !

After symmetry breaking, the interaction between the Higgs fields and the interaction bosons is going to generate mass terms :

$$\left(\frac{g_2}{2}\vec{\tau}.\vec{W}_\mu + \frac{g_1}{2}\hat{Y}\,B_\mu\right)\Phi = \begin{pmatrix} e\,A_\mu + \frac{g_2}{2}\frac{\cos 2\theta_w}{\cos\theta_w} Z_\mu & g_2\sqrt{2}\,W_\mu^+ \\ g_2\sqrt{2}\,W_\mu^- & -\frac{g_2}{2\cos\theta_w} Z_\mu \end{pmatrix} \frac{\lambda}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The Higgs, Englert and Brout mechanism

To generate the fermion masses, other interaction terms have to be added to the Lagrangian :

$$-\sqrt{2}\, f_l(\bar{R}_l \Phi^+ L_l + \bar{L}_l \Phi R_l) = -f_l\left[\ \overline{\psi_{l\ R}}\ (0, \chi(x)+\lambda) \begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}_L + \ (\overline{\psi_{\nu_l\ L}}, \overline{\psi_{l\ L}}) \begin{pmatrix} 0 \\ \chi(x)+\lambda \end{pmatrix} \psi_{l\ R}\right]$$

$$= -f_l(\lambda + \chi(x))\overline{\psi_l}\psi_l$$

where : $m_l = f_l \lambda$ is the mass term of the l fermion.

Lagrangian of free fields

free Lagrangian of W bosons

$$L_{CL} = -\frac{1}{2} W^{+}_{\mu\nu} W^{-\mu\nu} + M_W^2 W^{+}_{\mu} W^{-\mu}$$

$$-\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} M_Z^2 Z_{\mu} Z^{\mu}$$

free Lagrangian of Z boson

$$-\frac{1}{4} A_{\mu\nu} A^{\mu\nu}$$

free Lagrangian of photon

$$+\sum_{l} \bar{\psi}_{\nu_l} i \gamma^{\mu} \partial_{\mu} \frac{(1-\gamma^5)}{2} \psi_{\nu_l} + \sum_{l'} \bar{\psi}_{l'} (i \gamma^{\mu} \partial_{\mu} - m_{l'}) \psi_{l'}$$

free Lagrangian of massless neutrinos

free Lagrangian of fermions

with : $M_W = \dfrac{g_2 \lambda}{2} = 80.4\,\mathrm{GeV}$ $\quad M_Z = \dfrac{M_W}{\cos\theta_w} = 91.2\,\mathrm{GeV}$

$m_l = f_l \lambda$ and : $\lambda = 246\,\mathrm{GeV}$

Higgs boson Lagrangian

Kinetic energy term
of Higgs boson

Higgs boson
mass term

auto-interaction of
Higgs boson

$$L_H = \frac{1}{2}\partial_\mu \chi\, \partial^\mu \chi - h\lambda^2\chi^2$$

$$+\frac{1}{4}g_2^2[W^{+\mu}W^-_\mu + \frac{Z_\mu Z^\mu}{2\cos\theta_w}](2\lambda\chi + \chi^2) - h\chi^2(\lambda\chi + \frac{1}{4}\chi^2)$$

$$-\sum_l f_l \bar{\psi}_l \psi_l \chi$$

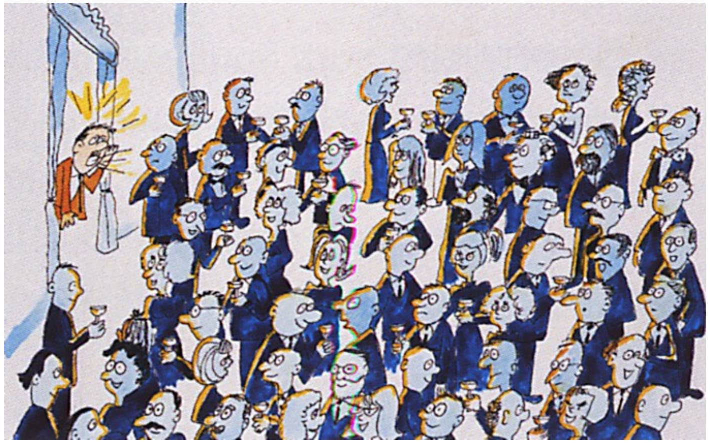
interaction term between Higgs boson and W and
Z bosons

Interaction terms between leptons and the Higgs boson

where : $m_\chi = \lambda\sqrt{2h}$ is the Higgs boson mass which is not directly predicted by the theory

or equivalently : $\mu = \frac{m_\chi}{\sqrt{2}}$ and $h = \frac{1}{2}(\frac{m_\chi}{\lambda})^2$

Le boson de Higgs

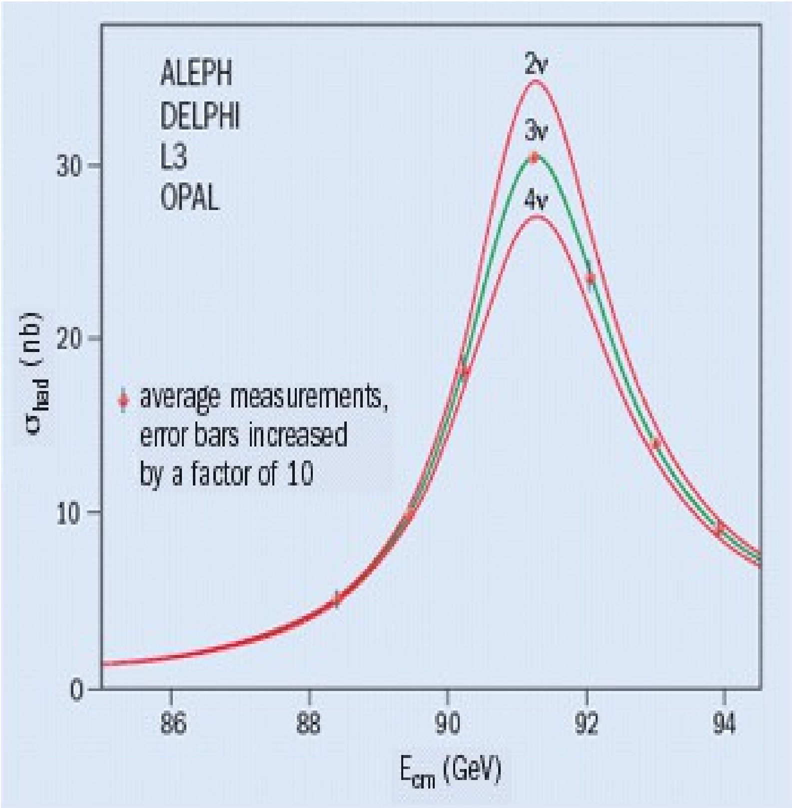


Excitation of the Higgs field

(LHC pp collisions)

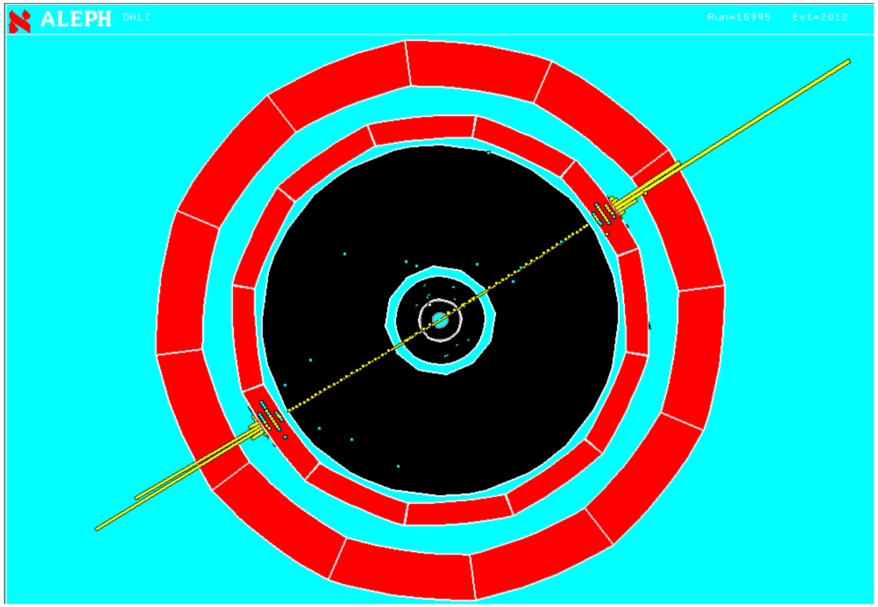
The perturbation propagates
, this is a Higgs boson,
which decays shortly after
into many channels

Z boson at LEP

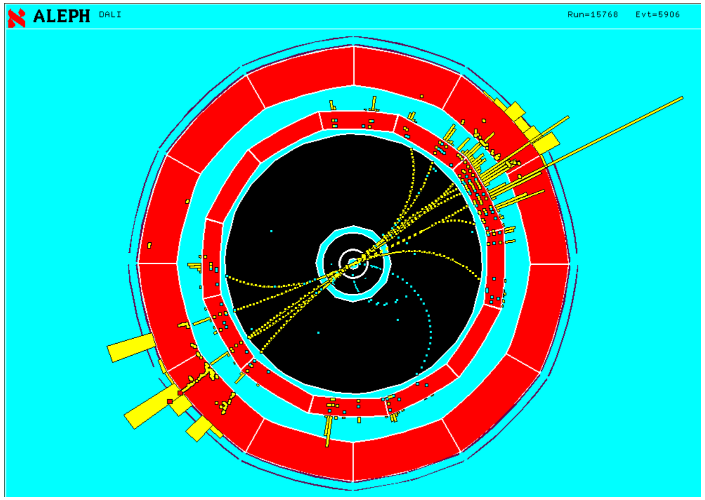


M_z = 91.1876 GeV (+/- 2.1 MeV)

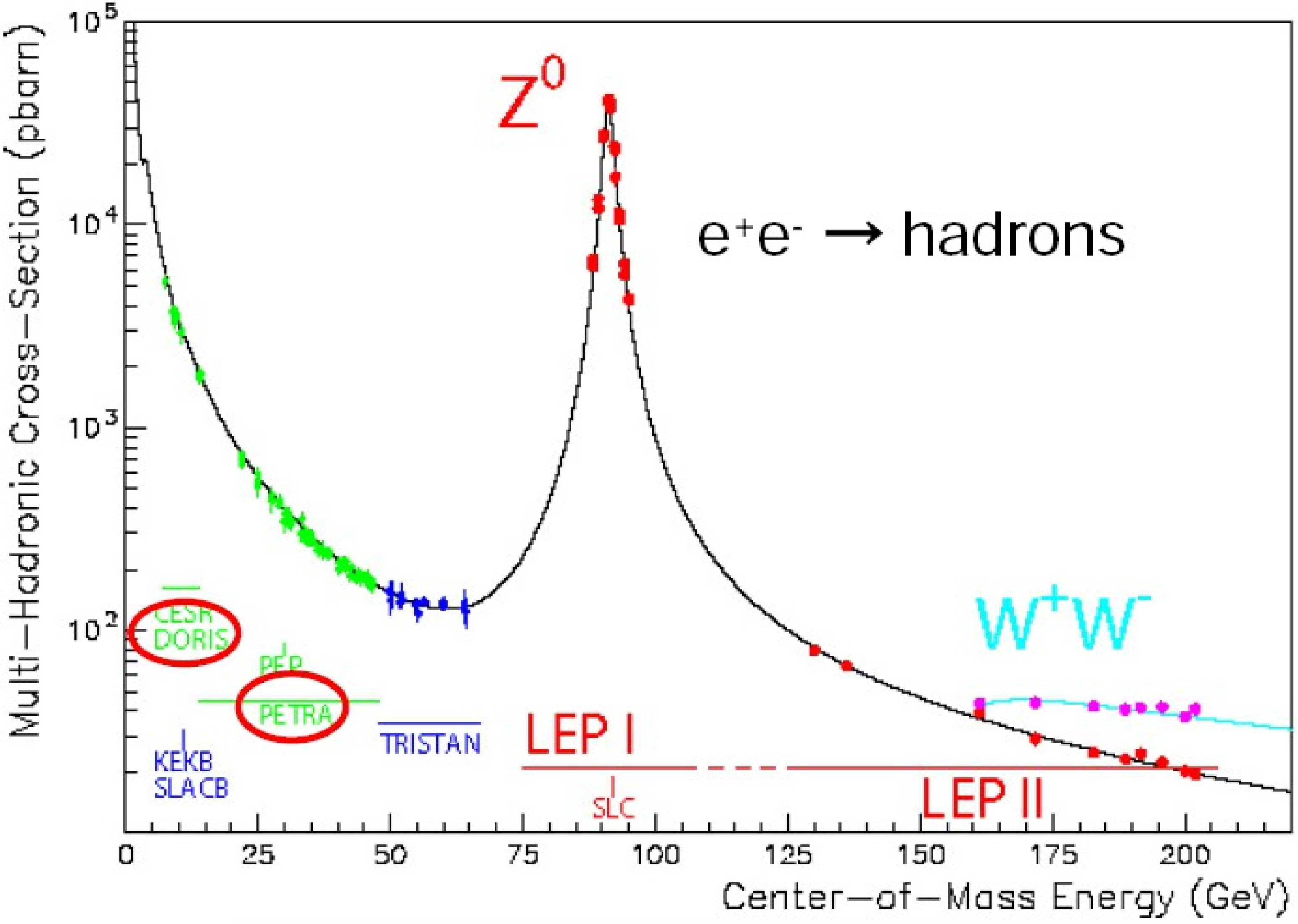
Less than three families with neutrino masses < 45 GeV



$Z \rightarrow e^- e^+$

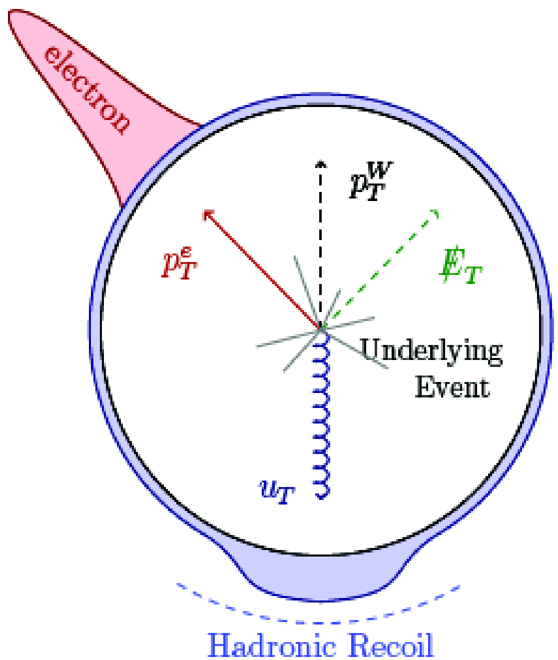


$Z \rightarrow q\bar{q}$



Hadron production in e^+e^- scattering

W Production at the Tevatron



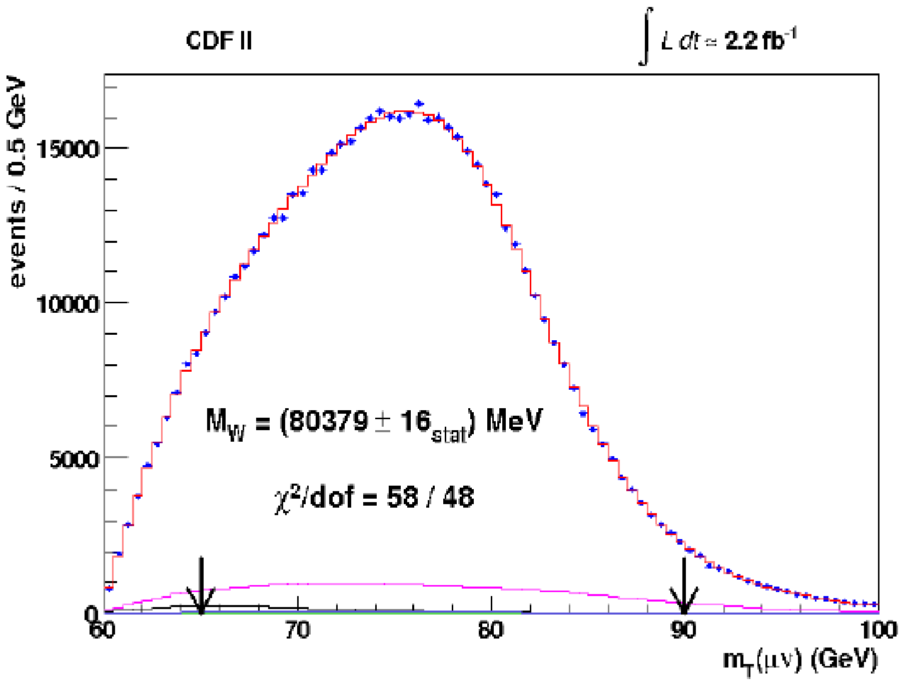
W: decays into single charged lepton + neutrino

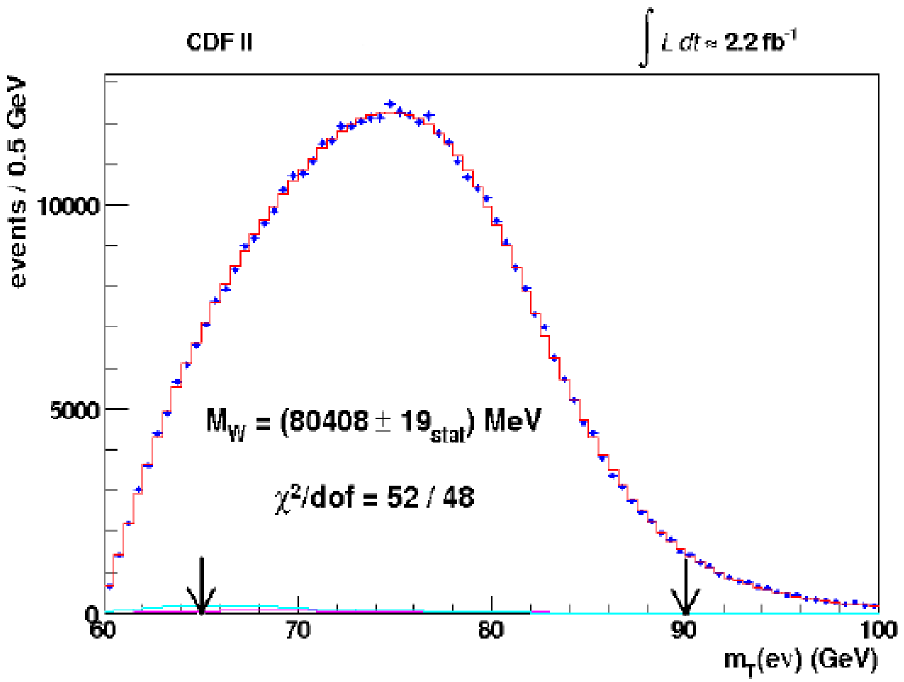
high p_T electron or high p_T muon

high E_T^{miss} (== neutrino p_T)

hadronic recoil momentum < 15 GeV

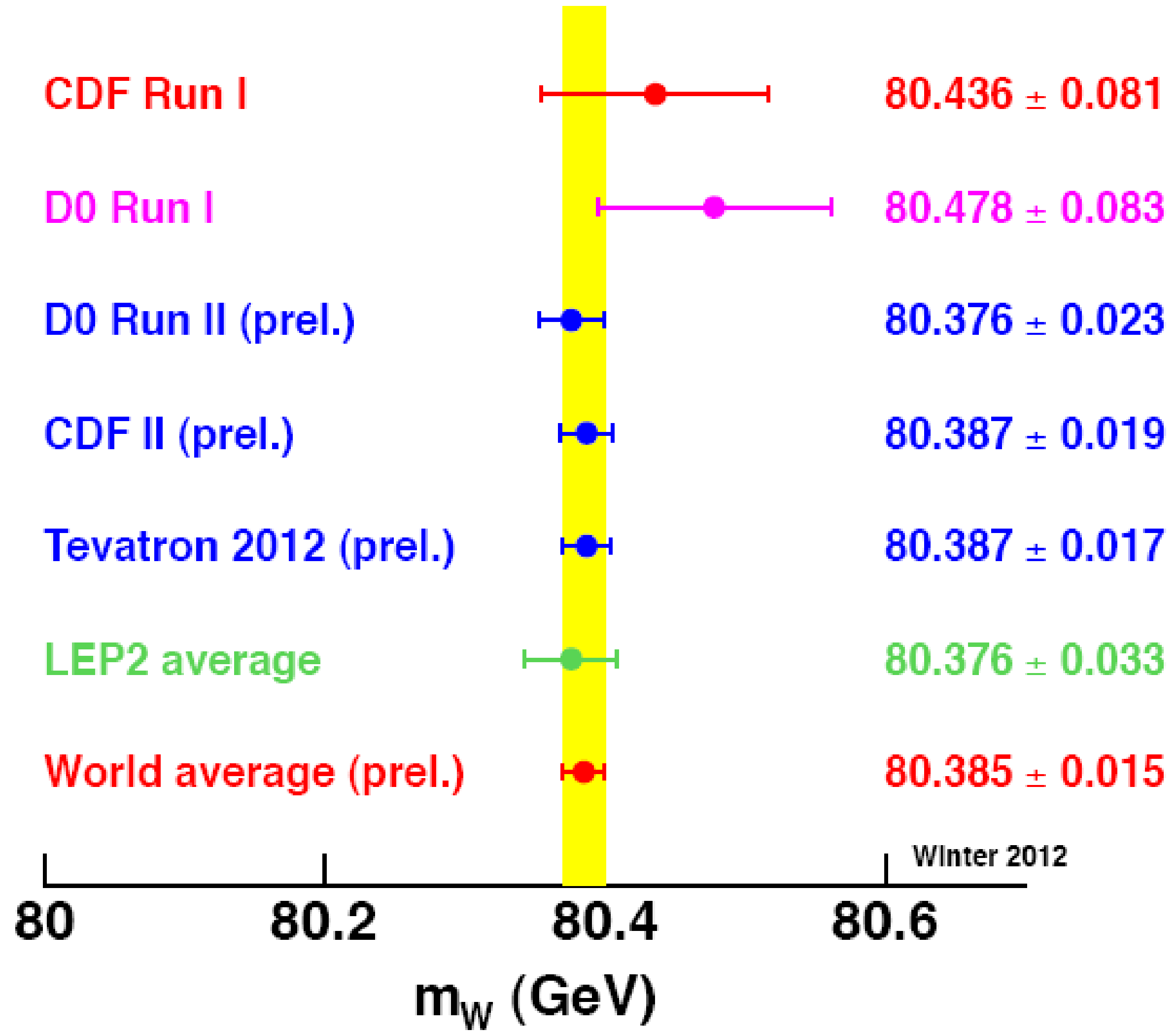
W transverse mass measurement at Tevatron





$$m_T = \sqrt{2\left(p_T^e\, p_T^\nu - \vec{p}_T^{\,e} \cdot \vec{p}_T^{\,\nu}\right)}$$

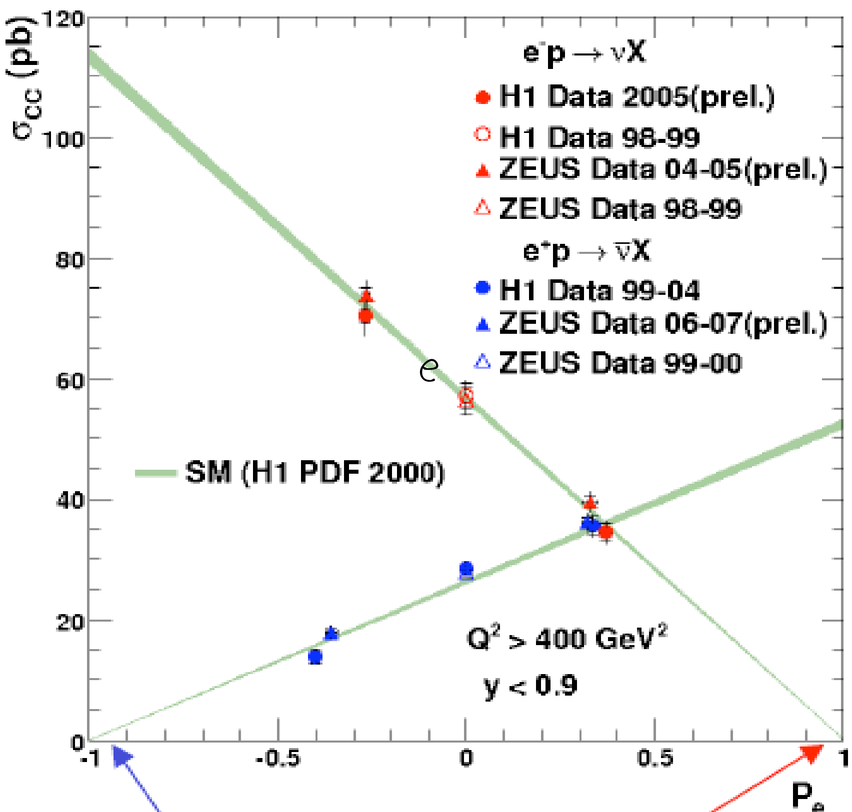
transverse mass of W (beware this definition is different from the one given p. 19)


CDF Run I
80.436 ± 0.081
D0 Run I
80.478 ± 0.083
D0 Run II (prel.)
80.376 ± 0.023
CDF II (prel.)
80.387 ± 0.019
Tevatron 2012 (prel.)
80.387 ± 0.017
LEP2 average
80.376 ± 0.033
World average (prel.)
80.385 ± 0.015
Winter 2012
80
80.2
80.4
80.6
m_W (GeV)

V-A structure of charged weak current measured on HERA collider at DESY in Hamburg

Charged Current (CC)

exchange of $W^\pm$ $(e^\pm p \to \nu X)$



Pc

left-handed e^+ and right-handed e^-

In the Standard Model and at high energy, left-handed helicity positrons and right-handed helicity electrons do not take part to the charged current interaction.